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# NUCLEAR STRUCTURE OF THE HEAVIEST BORON ISOTOPE

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*The two-neutron Halo of 19-Boronhas been investigated within this work. This investigation used the Microscopic Cluster Model (MCM). The main properties of Halo nuclei such as binding energy, radius, and deformation of the core have been calculated in this study. The $^{19}B$ has been defined in the shape of core-n-n. The $^{17}B$ is the core of the system. The feature of the three-body system depended on a structure and a deformation of the core. The core of $^{17}B$ hasn't been considered as an inert core but has some degree of freedom. This degree has a high influence on the structure of a three-body system. So we used the Microscopic Cluster Model (MCM). The main aim of this study is to expand using cluster model in a new version which is Microscopic Cluster Model.*

***Keywords:*** Halo-nuclei, $^{19}$B, Microscopic cluster model, neutron-halo structure, Wood-Saxon equation.

## Introduction

The facilities of radioactive nuclear-beam have made in considerations and brought to light a lot ofexciting phenomena in nuclearstructure physics, the study of atomic nuclei away from the line of the $\beta$-stability. The development of these facilities has proven to study the nuclear structure of neutron-rich atomic nuclei near to neutron-drip line (the threshold) for Fragmentation into core and neutrons in a very useful way.

In this study, of specific interest are the heaviest isotope of Boron, the two-neutron halo nucleus, $^{19}$B also the associated bound or unbound sub-system $^{18}$B, which are criticalto determining the $^{17}$B–n interaction inthe three-body model.Actually, the nuclear level structures of $^{18}$Bcan focus on the growth of the $\nu 2s_{1/2}$ level which can become degenerate in this area [1,2]. In terms of realizing the halo structure of a nucleus;calculations of total cross sections give the first indications [3]. The $^{19}$B measurements were started some time ago with very elevated energy (800 MeV per nucleon) and explained in a Glauber optical model analysis [4].

The matter radius of$^{19}$B was deduced 3.11±0.13 fm as compared to$^{17}$B, it is about 2.99±0.09 fm. This was slightly proposed that it may be defined as a $^{15}$B core and 4 neutrons. Enhanced electric-dipole (E1) is recognized just above the threshold of the two-neutron decay and E1strength of (E1)=1.64±0.06(stat)± 0.12(sys)$e$2 fm$^2$ for energies lower than 6 MeV. This feature, identified as E1 excitation, gives the first evidence that $^{19}$B has a structure of a two-neutron halo [5]. Actually, the boron $^{19}$B is a candidatefor in detailed studies of a probable multi neutron halo. Experimentally, this nuclide has little information: its binding energy is very low (but inexact) separation energy of two valence neutrons ($S_{2n} = 0.089^{+0.56}_{-0.089} MeV$) [4] and it's an enhanced interaction cross-section [4]. Then$^{18}$B isunbound, $^{19}$B has a Borromean property, where the $^{19}$B is bound asa three-bodysystem. These features are indicative of thestructure of a two-neutron halo. Anyway, being also loosely bound to four-neutron separation ($S_{4n} = 1.47 \pm 0.35 MeV$ ) [6]), $^{19}$B might slightly be described as “core-n-n-n-n” halo or as has a neutron skin [4,7].

The virtual state of even higher numbers of neutronsthe n-A body starts closing the threshold. A spectacular value of this trend obvious with the$^{17}$B isotope, where the virtual state of n-$^{17}$B is placed atthe excessivenearness of $^{18}$B threshold ground-state. This consequence is because the biggest value of the nucleus- neutron scattering length noticed until now, $a_S \sim -100$ fm [8]. Though, acceptance of the experiment and the limiteddid not permit to fix the scattering length a lower bound, and was determined an upper bound $a_S < -50$ fm only [8].

In spite of this experimental value uncertainty, the potentially massive n-$^{17}$B scattering length givesinterestingproperty to $^{19}$B, a two-neutron halonuclide [4] showing a weakly binding energy of core-n-n system [9]. Furthermore, because of its very weak binding energy (2n separation of $S_{2n} = 0.14 \pm 0.39$ MeV [10]), $^{19}$B has noexcited states.

Cluster models have a high range of applications in specific nuclear structure physics, from the light nuclei spectroscopy to low-energy reactions. When applying on exotic nuclei like halo nuclei or molecular states, the rest models unable to compete through cluster approaches up to now. For example, the shell-model has great problems to investigate cluster states. However, the difficulties related to this method make it appropriate basically to the light nuclei spectroscopy. Most microscopic cluster models or cluster models will likely be still commonly applied in the next works.

## 1. Theoretical Background

The $^{19}$B has been defined in the shape of core-n-n.The Hamiltonian of the $^{17}$B core defines a set of eigenstates $\phi_{core}$ and eigenvalues $\varepsilon_{core}$

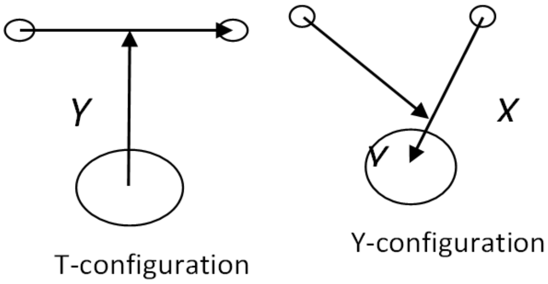

**Fig.1.** Jacobi coordinates for a three-body system.

$$\hat{h}_{core}(\xi_{core})\phi_{core}(\xi_{core}) = \varepsilon_{core}\phi_{core}(\xi_{core}) \tag{1}$$

The total wavefunction is

$$\Psi^{JM}(x, y, \vec{\xi}) = \phi_{core}(\xi_{core})\psi(x, y) \tag{2}$$

The Jacobi coordinates *(x, y)* well-defined as [11,12,13]

$$\rho^2 = x^2 + y^2 \text{ and } \theta = \arctan(\frac{x}{y})$$

$$\psi_k^{l_x l_y}(\theta) = N_k^{l_x l_y}(\sin\theta)^{l_x}(\cos\theta)^{l_y} P_n^{l_x+\frac{1}{2}, l_y+\frac{1}{2}}(\cos 2\theta) \tag{3}$$

The valence neutrons wave function is

$$\psi_{n,k}^{l_x l_y}(\rho, \theta) = R_n(\rho)\psi_k^{l_x l_y}(\theta) \tag{4}$$

So *ψ(x,y)* in eq(2) is $\psi(x, y) = \psi_{n,k}^{l_x l_y}(\rho, \theta)$

Thehyper spherical harmonics formalism as in Refs. [14,15]. The total Hamiltonian $\hat{H}$ ,is

$$\hat{H} = \hat{T} + \hat{h}_{core}(\vec{\xi}) + \hat{V}_{core-n1}(r_{core-n1}, \vec{\xi}) + \hat{V}_{core-n2}(r_{core-n2}, \vec{\xi}) + \hat{V}_{n-n}(r_{n-n}) \tag{5}$$

Thepotentials of deformed Wood-Saxon and a spin-orbit have been used [16, 17].

$$\begin{aligned}\hat{V}_{core-n}(r_{core-n}, \vec{\xi}) &= \frac{-V_0}{\left[1 + \exp\left(\frac{r_{core-n} - R(\theta,\phi)}{a}\right)\right]} \\ &+ \frac{-\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{core-n}}\frac{d}{dr_{core-n}}\left(\left[1 + \exp\left(\frac{r_{core-n} - R_{so}}{a_{so}}\right)^{-1}\right]\right)\end{aligned} \tag{6}$$

$$V_{n-n}(r_{n-n}) = -\frac{\hbar^2}{m^2c^2}(2l.s)\frac{V_{s.o}}{4r_{n-n}}\frac{d}{dr_{n-n}}\left(\left[1 + \exp\left(\frac{r_{n-n} - R_{so}}{a_{so}}\right)^{-1}\right]\right) \tag{7}$$

$$R = R_0\left[1 + \beta_2 Y_{20}(\theta, \phi)\right] \tag{8}$$

For more details about theoretical background is in [17,18]

## 2. Calculation

A valence neutron wavefunction has been described in eq. (5) whereas a core wavefunction is in eq. (2) ($\phi_{core}$). Equation (5) describes theHamiltonian of $^{19}$B. The Wood-Saxon potential has been applied to describe the central potential formula among the constituents along with s-p interaction as in eq. (6). The binding energy and radius of $^{19}$B and deformation of $^{17}$B have been calculated within this work. The central W-S potential relays on Quadrupole moment of $^{17}$B. From the figs. (2,3,4) the variation of nuclear potential with core deformation parameter $\beta_2$*is* clear and that means the shape of the core has high role in affecting on the nuclear force. In fig. (2) the nuclear potential of the s-state has ranged from (-5 to -17)MeV and the prolate shape has increased the potential widely. The potential between core and valence neutron in s-state in fig. (2) refers to a strong interaction between them and also refers to if the valence neutron in the s-state that means the core and valence neutron within the structure of shell model and it far from Halo shape. The figs. (3) and (4) more suitable with a halo structure.

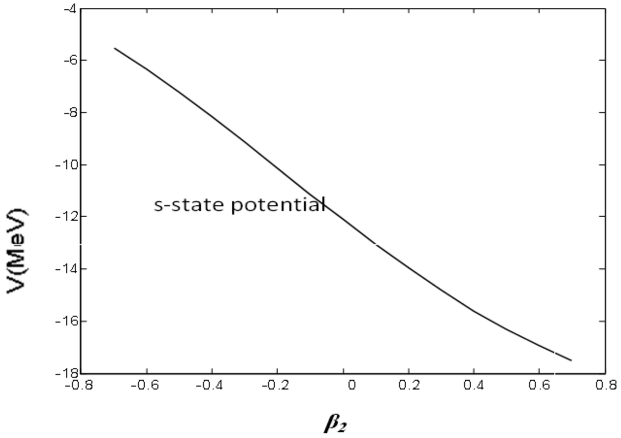

**Fig.2.** The s-state potential of $^{17}$B-n as function of deformation

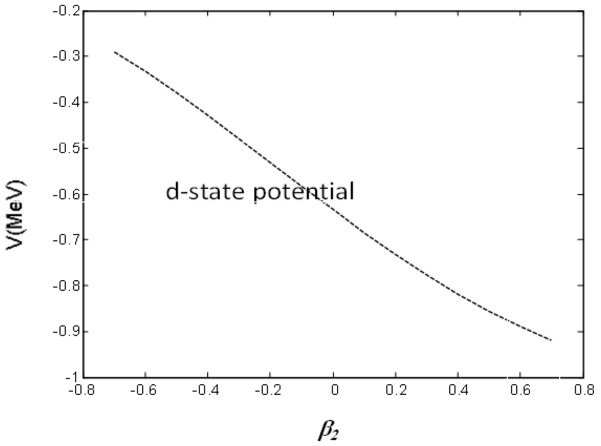

**Fig.3.** The d-state potential of $^{17}$B-n as function of deformation

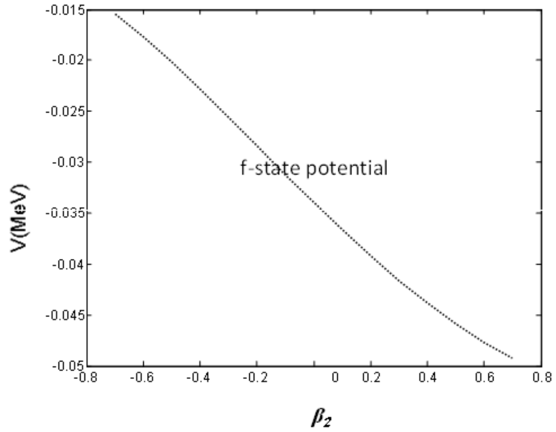

**Fig.4.** The f-state potential of $^{17}$B-n as function of deformation

The two-body energy state of $^{18}$B has been affected by the potential as seen in fig. (5). From the fig. (5), the two-body (the valence neutron and the core) has bounded energy. That means the $^{19}$B hasn't Borromean property. That isn't proved, because the bounded state of $^{18}$B is little, especially in the d-state as shown in fig.(5). So regarding s-state or f-state, we can say it has Borromean property. However, no experimental data indicate to the $^{19}$B has or hasn't Borromean property. But our calculation referred to no Borromean in $^{19}$B. The total angular momentum ($J^{\pi}$) of $^{17}$B is $J^{\pi}(^{17}B)=3/2^-$. The valence neutron bounded to the ground state ($^{17}$B) to form $J^{\pi}(^{18}B)$ or to excited state of ($^{17}$B). Also, the prolate shape of the core has a high influence in bound of two-body more than the oblate shape. So we can conclude from that, if the $^{18}$B bounded then the core has a prolate shape or if the $^{18}$B unbound then the core has an oblate shape.

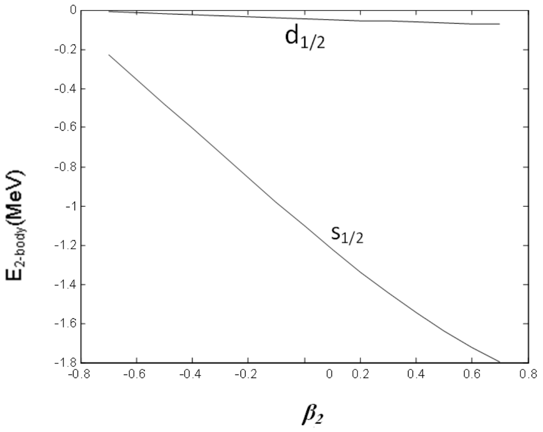

**Fig.5.** Two-body energy in $^{18}$ B as function of deformation with ground state of the core ($^{17}$B) $d_{5/2}$, $s_{1/2}$

The fig. (6) showed the three-body binding energy as a function of the deformed core ($^{17}$B). The two configurations ( T-configuration and Y-configuration) have been applied in the present calculation as shown in fig. (6). There isn't a big difference between the two configurations, especially with the oblate shape. Also, that meant the interaction between two valence neutrons hasn't high influence on the binding of the Halo nucleus. The features of $^{19}$B have been investigated with normalization and using experimental values. The radius of $^{19}$B is also diverse with deformation of the core as shown in the fig(7).

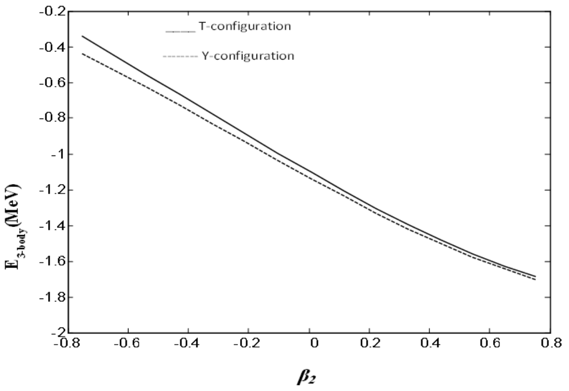

**Fig.5.** Energy of bound state of $^{19}$ Boron as function of deformation with ground state of the core $p_{3/2}$

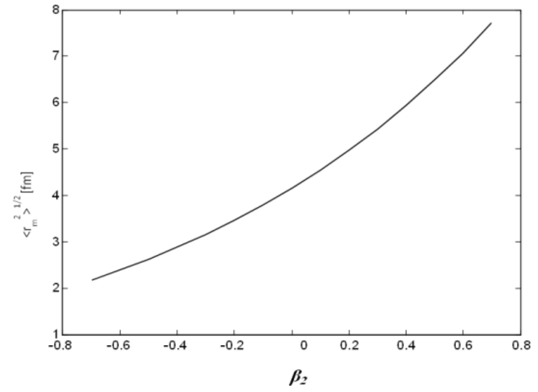

**Fig.7.** Radius of $^{19}$ Boron as function of deformation

As seen in the figures, the deformation has high effect on the properties of $^{19}$B. The$^{17}$B has 5 protons and 12 neutrons, therefore it is far from the closest closed shells (2,8,20) for both neutrons number and protons number. So it must have high deformation. Experimentally, there isn't evidence about the value of deformation. Regarding the experimental data, the core has the oblate shape with a high negative value as seen in the table (1).

**Table 1.** Experimental data of binding energy and matter radius of19-Boron

| Element | $E_{exp}$(MeV)[10] | $R_{exp}$(fm)[5] | quadrupole |
|---|---|---|---|
| 19-Boron | 0.14±0.39 | 3.11±0.13 | |
| 17-Boron | | | No evidence |

By normalization, the experimental value of binding energy from table (2) has been used in figure (6) to determine the ($^{17}$B) deformation which be (>-0.6). And this deformation referred to in figure (7) to determine the radius of (19-Boron) is about 3fm as in table (2).

**Table 2.** Experimental value of binding energy of 19-Boron with theoretical value (calculated) of matter radius of $^{19}$B and deformation parameter of $^{17}$B

| element | $E_{exp}$(MeV) | Deformation parameter | R(fm) Present work |
|---|---|---|---|
| $^{19}$B | 0.14±0.39 |  | 3 |
| $^{17}$B |  | >-0.6 |  |

Also by normalization, the experimental value of root-mean-square is (3.11±0.13fm) and has been used it in figure (7) to obtain the deformation parameter value (-0.6), which used in figure (6) to get the binding energy of 19-Boron which was 0.33MeV regarding figure (6) as seen in the table (3)

**Table 3.** Experimental value of matter radius of 19B with theoretical value (calculated) of binding energy of 19B i and deformation parameter of 17B.

| element | $R_{exp}$(fm) | Deformation parameter | E (MeV) Present work |
|---|---|---|---|
| $^{19}$B | 3.11±0.13 |  | 0.33 |
| $^{17}$B |  | -0.6 |  |

In the 19-Boron study, the transparent physical input has been used with the quantum mechanically rigorous at the same time to produce the main properties of this nucleus. In the 19-Boron case, we can consider the study has met the goal. The Pauli principle has been handled within this work. Improvement of the model can be done, and ideas of model development will be debated in the new study. For 19-Boron the study was able to produce a halo structure of this nucleus, and a huge difference between its radius and charge. The results are slightly similar to the experimental data. The study confirmed that $^{19}$B is loosely restricted, with the structure of the core-dineutron in agreement with most other theories. The results with Microscopic Cluster Model were better than with the Cluster Model and also we think the Wood-Saxon potential will be better than other potentials. The quadrupole moment was fairly big within this study. The Microscopic Cluster Model and Wood-Saxon potential show a very dominant core-dineutron system.

## Conclusion

The heaviest Boron's Isotopes (19-Boron) has been studied in the current work. The approach of this study is the Microscopic Cluster Model (MCM), which has more degree of freedom when dealing with a core. The 19-Boron is considered as a two-neutron Halo nucleus or a three-body system.It is built on an n-$^{17}$B real interaction which, supplemented through a realistic valence n-n potential, provides an acceptable explanation of $^{19}$B ground and states of resonant in terms of a three-body system$^{17}$B-n-n.The excitation and deformation of the core has a great effect on the structure of the three-body nucleus. So we can't deal with a core as an inert body as in Cluster Model.The calculations have confirmed the 19-Boron has the Halo structure. The calculations have confirmed the success of the microscopic cluster model and can use this model in other systems such as the molecules. The calculations referred to the two-body energy of $^{18}$B is bounded which means the $^{19}$B hasn't Borromean property, which made the $^{19}$B contrasted with all known Two-neutron Halo nuclei. From the results, we should in general use a better handling of the spin-orbit interaction to improve the results of the Halo structure.

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