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“Structure of convex existention prime jonson theories and classes of theories models”

Abstract
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Relevance of research. The model theory is a dynamically developing field of mathematics, which is at the intersection of such disciplines as mathematical logic, universal algebra, various sections of abstract algebra, topology, algebraic geometry. The famous specialist Jerome Kiesler in his reviews article [4] identifies two historically established trends in the model theory as the western and the eastern. At the same time he notes that the western model theories studies completing theories and the eastern one the Jonsson theories. The conventional names "western", "eastern" are associated with the geographical location of the founders such as the model theory in America, namely A. Tarski and A. Robinson. As we know, Robinson lived on the east coast of the United States, and Tarski lived on the western coast.

As practice shows, the results of the "eastern" model theory has got later than its "western" analog. If we take into account the fact that the apparatus for studying in completing theories is incomparably weaker today than the technical apparatus of correspondingly completing theories, we can assume that our interest in Jonsson theories is absolutely justified.

In the late of 1980s, the Kazakh mathematician T.G.Mustafin turned to this topic and defined the main goals and methods of working with Jonsson’s theories. He had got results which related with any Jonsson theories [6], [39-40] and so to specific examples of Jonsson theories [6, p. 135-197, 41]. The interest to the study of Jonsson's theory was continued by T.G. Mustafin and by A.R. Yeshkeyev and these studies can be traced to the following worksin the future. [12-13].

There is an important detail this is the set-theoretical aspect of the theorem on the existence of a semantic model the history of the study question of Jonsson’s theories. Although the difference between the western and the eastern directions is rather relatively, we note that the study of Jonsson’s theories refers to the problems of the "eastern" model theory.

In general, the Jonsson’s theories are incomplete, it would be interesting to consider the properties of existential formulas. In this case, the system of a topological space will not consist of open-closed sets, but of special closed sets under the additional completeness condition. The main instrument for investigating the Jonsson’s theory is the semantic method proposed by Professor T.G.Mustafin at the time, the essence of which is the translation of properties to the central replenishment to Jonsson prototype.

Recently, specialists in the model theory pay attention to the study of model-theoretic properties of structural problems of small models in the enrichment of the signature. And these enrichments must preserve certain properties of the studied objects, namely theories and their classes of models. As rules, in the case of the specific model-theoretic studies the models of completed properties of theories, it is very seldom transferred completely to the case of the study of Jonsson theories. This is due primarily to the fact that the Jonsson theories in general are not complete theories. And also, it should be noted that in the case of the study of Jonsson theories, specialists work mainly with the class of existentially closed models of these theories. Thus, projecting the main questions about small models from the theme of complete theories to the Jonsson theories, we will always be within the limits of the above limitations.

Theme of dissertation research in its content refers to the study of the model-theoretic properties of inductive theories. Inductive theory is a theory that distinguishes a fairly broad class of algebraic classical examples of theories that are widely used in all areas of mathematics. The class of Jonsson theories is a fairly wide subclass of inductive theories that define almost all basic algebraic examples, such as groups, Abelian groups, fixed characteristic fields, Boolean algebras, linear orders. But nevertheless, this class is quite wide, and with a poor technique for studying the Jonsson theories, it would be reasonable to find some other natural limitations in the class of theories under study. In this thesis, such a natural limitation will be the concepts of convexity and existential primeness. We will deal with the concepts of convexity and existential prime in the class of Jonsson theories. The notion of convexity of the theory was introduced in A. Robinson's book [16, p. 118-123]. The concept of existential primeness was introduced by Yeshkeyev A.R. [13]. The concept of the existential prime of the theory is a natural additional condition in the study of the properties of the Jonsson theory. By virtue of the inductivity of the theory, the class of existentially closed models of the Jonsson theory is always not empty, but the class of algebraically prime models of the Jonsson theory under consideration can also be empty. In addition to the concepts of convexity, there is the notion of strong convexity of the theory and this concept was also introduced in A. Robinson's book. A strongly convex theory by definition has a core model, which in its turn is algebraically prime, but it is not necessarily existentially closed. The most vivid example showing that there are many strongly convex theories is an example of group theory. This example is characterized by the fact that this is an example of an imperfect Jonsson theory. In the case of the theory of abelian groups, we have an example of a perfect strongly convex Jonsson theory. The perfection of the Jonsson theory and some fragment in this theory does not depend on each other, there is a perfect Jonsson theory, but an imperfect fragment in this theory and, conversely, there is a perfect fragment in the imperfect Jones theory. In this case, the strongly convex perfect Jonsson theory is existentially prime.

Thus, we can conclude that the natural restriction of the class of Jonsson theories to the class of strongly convex perfect Jonsson theories is justified, but separately one can consider the class of existential-prime perfect Jones theories.

The main part of the thesis is a model-theoretic study of the properties of Jonsson’s sets by a fixed convex existential-prime perfect Jonsson theory. The Jonsson’s sets were defined earlier by A.R. Yeshkeyev and was begun the work of the research [30, p. 53-62; 31, p.217; 32, p.8]. The idea of the Jonsson’s set goes back to the concept of a basis of a linear subspace. By the way, the theory of linear spaces over a fixed field is an example of a perfect Jonsson theory. In [43, p. 48-56], questions of countable and uncountable category was examined for the Jonsson’s sets of prime the Jonsson theories. Special closures of these sets will be considered under such fragments. The study of such properties lies in the mainstream of the study of Jonsson’s theories. The Jonsson theories form is a fairly wide class of theories, which are examples of the main classical examples of algebras. On the other hand, one can easily come to the Jonsson theory with the help of natural enrichments, like whine making or morphism (skulemization or moralization) from an arbitrary complete theory [4]. But, the Jonsson theories are in general in completed and the research apparatus of such theories is insufficient for the modern realities of the model theory. Thus, the development for the study of Jonsson sets is an actual problem.

In general, the thesis consists of two chapters, they are interrelated. The first chapter is devoted to the basic concepts of the theory of models and reflects almost all the basic concepts associated with the Jonsson theories. These include concepts such as model completeness, existential closedness, convex theories, axiomatization of convex theories, strong substructures, types of atomic models, properties of amalgam and compatible, existentially closed models, different types of morphisms between models, different types of companions, quantifier elimination.

The second chapter of the theses deals with the Jonsson sets, the stability of the Jonsson’s theories with relative to existential types, forking in the class of perfect Jonsson theories. Various types of companions, various ways of classifying fragments of Jonsson’s sets in existentially prime convex perfect Jonsson theories with relative to the categoricity and cosemantic character of the closures of these sets. Also, there are examined the syntactic and semantic properties of their models through the concept of cosemantic for fragments under consideration of the Jonsson sets. The Central type of theory relating to some of Jonsson subsets of the semantic model of the convex existential primeJonsson theory is determined. The properties of an elimination of quantifiers and the countable and uncountable categorical fragments of the # - companion of Jonsson subset of the Jonsson theory' and the lattice of fragments of Jonsson sets are considered.

Objective. The main goal of the dissertation research is to obtain a description of the syntactic and semantic properties of the structure of convex existentially prime Jonson's theories and their classes of models.

Object and subject of study. The objects of the study are the Jonsson theory, also the class of its existentially closed models.

Research methods. We classify all the classical methods of research in the theory of models as well as the semantic method that has been developing lately. The essence of this method consists in transferring the properties of the first order of the predicate calculus from the center of the Jonsson's theory under consideration to the theory itself. A semantic method is also applied to the study of Jonsson's sets and their convex fragments.

Scientific novelty. In this dissertation was studied the restriction of the Jonsson's theories to a class of existentially prime convex theories which is an entirely new approach, the class which has not been studied before.

Theoretical and practical value of the work. The results have a theoretical nature and can be applied in further studies in the fields of universal algebra and the model theories.

Approbation of the results. According to the results of the thesis, reports were made at international conferences and seminars; including at the third international conference on analysis and applied mathematics. Institute of Mathematics and Mathematical Modeling (September 07-10, 2016) - Almaty, 2016, International Scientific Conference "Actual Problems of Mathematics, Mechanics and Informatics" (December 9-10, 2016) - Karaganda, 2016, the annual scientific April conference of the Institute of Mathematics and Mathematical Modeling dedicated to the Day of Science (7,8 April 2017) .- Almaty, 2017, international conference "Actual problems of pure and applied mathematics" dedicated to the 100th anniversary of the birth Academician Taymanov A.D. (August 22-25, 2017) .- Almaty, 2017, the First Congress of the Mathematical Society of Turkic-speaking countries (October 2-5, 2017) .- Astana 2017, for the whole academic year was held a scientific seminar of the department of algebra, geometry and mathematical logic named after Prof. Mustafin T.G. "Selected issues of model theory", as well as reports in the laboratory of mathematical logic at the" Institute of applied mathematics " Of the Committee of science of the Ministry of education and science of the Republic of Kazakhstan Karaganda.

Publications on the results. The main results of the dissertation research were published in 23 papers. Of these, 1 in the magazine included in the Scopus database, 5 in publications recommended by the Education and Science Control Committee of the Ministry of Education and Science of the Kazakhstan Republic, 4 in the far abroad, 7 in the materials of international conferences in the Republic of Kazakhstan, 5 in materials of foreign international conferences and 1 monograph. In the works performed with the co-author, the contribution of each of the co-authors is equal.

Provisions **to be defended.**

The following basic provisions of the work are submitted for defense:

1) some subsets of the semantic model of the convex existentially prime perfect Jonsson theory are studied;

2) criterion for countable and uncountable categorical strongly minimal Jonsson fragment of theconvex existentially prime perfect Jonsson theory was found;

3) the properties and connection of the companions of the convex existentially prime perfect Jonson theory's fragment are studied;

4) the properties and relationships with koumantou between a fragment of a subset of the semantic model of the considered theory;

the properties and connection with the cosemanticness of a fragment of a subset of the considered theory's semantic model are studied;

5) the connection of properties of the lattices of existential formulas of Jonsson set and the properties of central completion of considered theory was found;

6) the stability of the central type of the considered theory and the equivalence of a forcing companion under perfect and convex existential completeness are shown;

7) the properties of countable and uncountable categoricity and elimination of # -companion fragment quantifiers of the semantic model of the higher theory under consideration are studied;

the properties of a countable and uncountable categoricity and elimination of quantifiers of fragments of the #- companion of the considered theory's semantic model were studied;

8) the lattices of existential formulas for fragments of Jonsson set are studied.

Structure and scope of work. The dissertation work consists of an introduction, two sections, conclusion, definition, designation and abbreviation, a list of used sources from 68 works. The work is set out on 92 pages. The numbering of formulas, theorems, lemmas and remarks in the sections is three – digit; the first number means the section number, the second number means the number of the subsection, the third-the proper number of the formula, theorems, lemmas and remarks, respectively, within the subsection.

Summary of the main part. In the first section we give some auxiliary facts and statements.

Consider the theory of T a countable L be a first-order language.

Definition 1.5.5. [13, p.158].The model A of the theory T is called T - existentially closed if for any model B and any existential formula $\varphi(\bar{x})$ with constants from A, performed $A \models \exists \bar{x} \varphi(\bar{x})$, on condition that the A is submodel of B and $B \models \exists \bar{x} \varphi(\bar{x})$.

The model M of the theory T is called algebraic prime if it is isomorphically embed in any model of this theory T [1 chapter, p. 15]..

Definition 1.1.4. [12, p. 95]. The theory T is called convex if for any of its model $\mathfrak{A}$ and for any family $\{B_i | i \in I\}$ of submodels of $\mathfrak{A}$, which are models of T, the intersection $\bigcap_{i \in I} B_i$ is a model of T.

If it is assumed that this intersection is not empty. If this intersection is never empty, then the theory is called strongly convex.

Let Jonsson theory T be complete for existential sentences and C be its semantic model .

Definition 2.1.6. (Yeshkeyev A.R.) We say that the set X is Σ are definable if it is definable by the some existential formula.

The set X is called Jonsson in the theory T if it satisfies the following properties:

1. X is a Σ are definable subset of C;
2. $Dcl(X)$ is a carrier of some existentially closed submodel C.

Let X Jonsson set in the theory T and M be an existentially closed submodel of X the semantic model C, where $dcl(X) = M$.

Definition 2.2.1. (Yeshkeyev A.R.) We say that all $\forall\exists$-conclusions of arbitrary set create Jonsson fragment of this set, if the deductive closure of this $\forall\exists$-conclusions is Jonsson theory.

Definition 2.2.2. (Yeshkeyev A.R.) The inductive theory T is called the existentially prime if

1. T has at least one algebraic prime (AP) model, (T_{AP}-class of all algebraic prime model of T);
2. $T_{AP} \cap E_T \neq 0$.

In the framework of the above definitions Jonsson's sets fragmentsare considered for the class of convex existentially prime perfect Jonsson theories. In turn, will entail a number of new statements of problems, for example, the refinement of first-order properties for the companion fragments (used to prove Theorem 2.2.5), the refinement of the Lachlan-Baldwin theorem on uncountable categoricity in the framework of this new topic (used to prove the theorem 2.2.6).

Let X Jonsson set in the theory T and M is existentially closed submodel of the semantic model C, considered by the Jonsson theory T, where $dcl(X) = M$.

$Th_{\forall\exists}(M) = T_M$, T_M is the Jonsson fragment of Jonsson set *X*, T_M^* is the center of T_M.

Theorem 2.2.5. Let *T* is the perfect existential prime convex Jonsson theory. Then the following conditions are equivalent:

1) T_M^* is ω- categorical;

2) T_M is ω- categorical.

Definition 2.2.8. [13, p.152]. Let $A, B \in E_T$ and $A \subset B$. Then *B* is algebraically prime extension *A* to E_T, if for any model $C \in E_T$ so that if *A* is isomorphically embedded in *C*, then *B* is isomorphically embedded in *C*.

Theorem 2.2.6. Let *T* be a perfect existential prime convex Jonsson theory. Then the following conditions are equivalent:

1) T_M^* is ω_1 - categorical;

2) any countable model E_{T_M} has a prime algebraic extension in E_{T_M}.

In the third section, is considered the properties of variousfragments companions of convex existentially prime perfect of Jonson's theories, their relation to each other, also the connection with the original theory is under consideration. It is proved the equivalence of the concepts of completeness and model completeness in the class of considerable subclasses of the class of all Jonsson's theories. The concept of a companion theory was introduced by A. Robinson [4, p.156-157] and played an important role in the study of inductive theories. Various companions have been previously considered for the Jonsson's theories in the works of Yeshkeyev A.R. So, as in the study of the closure of the Jonsson's subset a new class semantics model of theories were under consideration, there can also be various companions, we will consider analogues of results for above subclasses of Jonsson's theories with the help of Jonsson sets.

Theorem 2.3.2. Let *T* is the perfect existential prime convex Jonsson theory. Then the following conditions are equivalent:

1) T_M is perfect;

2) T_M^* is $\forall\exists$ - axiomatizable.

Theorem 2.3.3. Let *T* is the perfect existential prime convex Jonsson theory. Then the following conditions are equivalent:

1) T_M is perfect;

2) $T_M{}^* = T_M{}^0$.

Theorem 2.3.4. Let *T* is the perfect existential prime convex Jonsson theory. Then the following conditions are equivalent:

1) T_M is perfect;

2) T_M has a model companion.

By analogy with well-known fact from [12, p.30] can state the following:

Lemma 2.3.1. If the $T_M{}^{\#}$ is companion of is the perfect existential prime convex Jonsson theory T_M and $T_M{}^M$ is model companion of T_M, then $T_M{}^{\#} = T_M{}^M$.

It is easy to check the following:

Lemma 2.3.2. Let T_1 and T_2 are perfects existentials primes convex Jonsson theories. Then the following conditions are equivalent:

1) T_{M_1} and T_{M_2} are mutually model consistent;

2) $T^{\#}_{M_1} = T^{\#}_{M_2}$.

One of the results of this paper is the following theorem, which is relevant to Lindstrem's theorem on a model completeness [4, p.164].

Theorem 2.2.6. Let T is the perfect existential prime convex Jonsson theory. The following conditions are equivalent:

1) T_M is complete theory:

2) T_M is model complete theory.

In the fourth section it is considered various ways of classifying fragments of the Jonsson's sets in convex existentially of prime perfect Jonsson's theories with relatively cosemantic closure of these sets. Also, it will be considered the syntactic and semantic properties of their models through the concept of cosemantic for fragments of the considered Jonsson's sets. The considered classification is a cosemanticity between subsets fragments of the semantic model of the considering theory.

Definition 2.4.3. (Yeshkeyev A.R.) The models A and B of signature σ are called cosemantic (symbolic $A \rhd\lhd_J B$) if for any Jonsson theory T_1 such that $A \models T_1$ there is Jonsson theory T_1, which is cosemantic with T_1 such that $B \models T_2$. And conversely.

Lemma 2.4.3. Let T be a perfect existential prime convex Jonsson theory in a language L. Let X_1, X_2 are Jonsson sets in theory T. Then true the following implications:

$$M_{X_1} \equiv M_{X_2} \Rightarrow M_{X_1} \equiv_J M_{X_2} \Rightarrow M_{X_1} \rhd\lhd_J M_{X_2}.$$

Problem. To get description of cosemantic Jonsson algebras from the above list of Jonsson algebras.

Corollary 2.4.1. Suppose $T_{M_{X_1}}$ is the fragment of X_1, $T_{M_{X_2}}$ is the fragment of X_2 where X_1, X_2 - Jonsson sets of the perfect existential prime convex Jonsson theory T. And C_1 is the semantic model of $T_{M_{X_1}}$, C_2 is the semantic model of $T_{M_{X_2}}$. Then, if $\left(T_{M_{X_1}}\right)_\forall = \left(T_{M_{X_2}}\right)_\forall$, then $T_{M_{X_1}} \rhd\lhd_J T_{M_{X_2}}$.

Theorem 2.4.1. Let $T_{M_{X_1}}$ and $T_{M_{X_2}}$ fragments of Jonsson sets X_1, X_2 in the perfect existential prime convex Jonsson theory T. Let C_1 be a semantic model of $T_{M_{X_1}}$, C_2 be a semantic model of $T_{M_{X_2}}$. Then the following conditions are equivalent:

1) $C_1 \rhd\lhd_J C_2$,

2) $C_1 \equiv_J C_2$,

3) $C_1 = C_2$.

Theorem 2.4.2. Let M_{X_1}, M_{X_2} be existentially closed submodels of a semantic model of a perfect existential prime convex Jonsson theory T such that X_1, X_2 are Jonsson sets in the theory T. Then the following conditions are equivalent:

1) $M_{X_1} \rhd\lhd_J M_{X_2}$,

2) $\forall\exists(M_{X_1}) \rhd\lhd_J \forall\exists(M_{X_2})$.

In the fifth section itis defined the concept of the central type of the theory with fraction to some Jonsson's subset of the semantic model of a convex existentially simply perfect Jonsson's theory. It is shown the equivalence of the stability of the central type such a theory with its forcing companion under the condition of exemplarity and convex, existential completeness.

The above-mentioned definitions can be easily shown for the Jonsson theory of $\exists$-sentences in the framework of existentially prime perfect complete convexity. We obtained the following results

Let us consider the concept of stability in enrichment by the Jonsson set A.

Theorem 2.5.5. Let λ be an arbitrary infinite cardinal, T convex existentially prime perfect complete for $\exists$-proposals the Jonsson theory. Then the following conditions are equivalent:

1. $(T^*)^F$ λ is stable in the complete theory, where $(T^*)^F$ – forcing companion of theory T^* in enriched signature;
2. T^* - λ is stable in the complete theory.

In the sixth paragraph we considers such model-theoretic properties as countable and uncountable categoricity and the properties of elimination of # - companion quantors of the perfect fragment of a certain Jonsson subset of the fixed Jonsson theory's semantic model. The interest in studying the model-theoretic properties of the # - companion goes back to the problems that were formulated when studying inductive theories. This class of theories was well studied in the literature [4] and the main inspirer of the this subject study was one of the founders of the theory of models Abraham Robinson.

In this paragraph we focus our attention on the properties $Fr^{\#}$ (A) - # - companion of the Jonsson set fragment A of the perfect fixed Jonsson theory T.

Theorem 2.6.3. Let A be the Jonsson set and $Fr(A)$ is the perfect existential prime convex Jonsson theory. The following conditions are equivalent:

1) $Fr(A)$ is complete:

2) $Fr(A)$ is model complete.

Theorem 2.6.8. Let us suppose that $Fr(A)$ a Jonsson fragment, which is an existentially prime perfect Jonsson theory complete for the existential sentences of the Jonsson universal theory, for which R_1 is satisfied. Then the following conditions are equivalent:

1) theory $Fr^{\#}$ (A) ω_1 is categorical,

2) any countable model of $E_{Fr(A)}$ has an algebraically prime model expansion in $E_{Fr(A)}$.

Let us now study the model-theoretic properties of the #-companion of the perfect Jonsson fragment. Let *T* be the Jonsson theory of a countable language *L*, and let *A* be the Jonsson subset of the semantic model *T*, $Fr(A)$ - the fragment of the Jonsson set *A*. Let $E_n(Fr(A))$ be a distributive lattice of equivalence classes $\varphi^{Fr(A)} = \{\psi \in E_n(L) | Fr(A) | - \varphi \leftrightarrow \psi\}$, $\varphi \in E_n(L)$, $E(Fr(A)) = \bigcup\limits_{n<\omega} E_n(Fr(A))$.

Now we consider the fragment of the Jonsson set A, which is complete for $\exists$ -proposals.

Theorem 2.6.9. Let $Fr(A)$ be the perfect fragment of the Jonsson set A, $Fr^{\#}(A)$ its #-companion. Then

1) $Fr^{\#}(A)$ admits the elimination of quantifiers if and only if each $\varphi \in E_n(Fr(A))$ has a quantifier-free complement;

2) $Fr^{\#}(A)$ is positively model complete if and only if each $\varphi \in E_n(Fr(A))$ has an existential complement.

In the following theorem, in terms of the $E_n(Fr\ (A))$ lattice of existential formulas, necessary and sufficient conditions for the perfectness of the Jonsson theory T are found.

Theorem 2.6.10. Let $Fr(A)$ be perfect fragment of the Jonsson set A, $Fr^{\#}(A)$ its #-companion. Then the following conditions are equivalent:

1) $Fr(A)$ is perfect;

2) $E_n(Fr(A))$ is weak complemented;

3) $E_n(Fr(A))$ is Stone algebra.

There're consideredthe theoretical-model properties of the Jonsson's theoriesin the seventh section. In particular, it'sconsidered the lattice of special formulas. In the sixth paragraph, was under consideration the model-theoretic properties of the fragment of the Jonsson'ssets. In particular, the lattice of existential formulas. When studying complete theories, one of the main methods was to use the properties of the topological space $S_n(T)$. In the language of ultrafilters of the Boolean algebra $F_n(T)$, where T is a fixed theory,are studied such classical concepts of the model theory as model and theory stability, model saturation, model homogeneity, model diagrams, etc. Since the existential formula is not closed at all relatively to Boolean operations in the topological spaces of existential types are significantly different from all cases. It is clear, that such an approach (the restriction of $F_n(Fr(X))$ to $E_n(Fr(X))$) is a generalization of the case when we deal with the full theory. Since the fragment of the Jonsson's sets as a whole is incomplete, it would be interesting to consider the properties of the lattice of existential formulas.

In following we shall have to deal with a fixed convex perfect existentially prime Jonsson theory T complete for existential sentences and its semantic model is a C. Let $X \subseteq C$ and X be the Jonson set.

Let us note that since a theory which is complete for existential sentences satisfies the joint embedding property, but the converse is not true, we see that the condition of existential-completeness in our theorems cannot be eliminated.

Hereinafter all considered theories will be complete for the Σ- sentences.

Theorem 2.7.1. Let $Fr(X)$ is be a fragment of Jonsson set, $Fr^*(X)$ be the center of the fragment of Jonsson set $Fr(X)$. Then

1) $Fr^*(X)$ admits elimination of quantifiers if and only if every $\varphi \in E_n(Fr(X))$ has quantifier-free complement;

2) $Fr^*(X)$ is a model-complete if and only if every $\varphi \in E_n(Fr(X))$ has a existential complement.

Theorem 2.7.2. Let $Fr(X)$ be a fragment of Jonsson set X. Then the following conditions are equivalent:

1) $Fr(X)$ is a perfect;

2) $E_n(Fr(X))$ is weakly complemented;

3) $\varphi \in E_n(Fr(X))$ is a Stone lattice.

Theorem 2.7.3 Let $Fr(X)$ be a fragment of Jonsson set. Then the following conditions are equivalent:

1) $Fr^*(X)$ is a theory;

2) each $\varphi \in E_n(Fr(X))$ has a weak quantifier-free complement.

The thesis was carried out at the chair of "Algebra, mathematical logic and geometry named after Professor T.G. Mustafin" Karaganda State University.