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**The model-theoretic properties of companions of fragments
of Jonsson sets**

Annotation of the dissertation for the degree of Philosophy Doctor (PhD) in the
specialty 6D060100 - Mathematics

Actuality of the research theme. The issues reflected in the topic relate to the classical issues of model theory. The concept of companions of a fixed theory was introduced and considered by A. Robinson. Companions are of various types and, as further studies have shown, there is a nontrivial connection between them and their description is associated with the semantic properties of the class of models of the considered theory. As a rule, the study of companions was associated with inductive theories. The latter include basic examples of classical algebras, such as various types of rings, groups, lattices and polygons. In the future, with the development of the apparatus of model theory, structural questions of classification of elementary model theories were intensively studied in the language of the stability properties of formula subsets of the considered model. It should be noted that this technique of studying companions initially implies the completeness of the theory. In the class of incomplete inductive theories, the central place is occupied by Jonsson theories, which satisfy the natural algebraic requirements, as a property of joint embedding and amalgamation. The Jonsson theories are theories of the following classical algebras: fields of fixed characteristic, Abelian groups, groups, Boolean algebras, unars.

Today, perfect Jonsson theories are well-studied. Companions of perfect Jonsson theories preserve many of the properties of the original Jonsson theory: firstly, they all coincide, and secondly, they preserve such important model-theoretical properties as the Jonsson theory and, with Jonsson stability, the stability of its center as one of the companions. In fact, in the perfect case, the center is the only companion. An imperfect case has not yet been completely studied, so a striking example of an imperfect Jonsson theory is the Jonsson group theory, which does not have a model companion, but there is a forcing companion in the countable case. Moreover, in the class of imperfect Jonsson theories, perfect Jonsson subclasses. An example is the class of all Abelian groups, which is perfect and at the same time it is a subclass of the class of all groups. The theory of abelian groups has a model companion.This model companion is a class of all algebraically closed groups.

The next stage of research is the study of special formula subsets of the semantic model of the considered Jonsson theory. In this case, the role of formula sets is played by Jonsson sets, the definable closures of which coincide with the support of some existentially closed submodel of the semantic model of the considered Jonsson theory. At considering of a fragment of given set, namely, the deductive closure of all universally existential sentences true in this definable closure, we have the opportunity to consider the model-theoretic properties of the

companions of this fragment.

The study of the properties of companions of Jonsson fragments makes a lot of sense, both from the position of problems of the classical model theory and new approaches of the study of incomplete Jonsson theories. The existence of companions is not always obligatory, therefore, the ability to find the conditions for their existence is an interesting and actual problem. The main research issue is the study of model-theoretical properties of companions of fragments of Jonsson sets. Also, the companions of the Jonsson theory and their relationship, model-theoretic properties of fragments of definable closures, a description of strongly minimal sets on the pregeometry of a semantic model of a fixed Jonsson theory, and the properties of lattices of existential formulas of Jonsson fragments are considered in this work.

The study of the structural issues of a theory and, accordingly, its models is an actual task of model theory. At the study of the properties of model, it is necessary to know the properties of its elements. Therefore, we must go to some restrictions of the properties of models and their subsets, since in the general case such task for incomplete theories seems quite impossible. As is known in the general case, the axioms of Jonsson theories satisfy almost all the basic types of algebraic objects, but they, unfortunately, are not complete in a logical sense and the developed technique of proofs and the methods and concepts used from model theory are usually given for complete theories and, accordingly, are not work in the case of Jonsson theories. Therefore, the development of the research apparatus and the receipt on this basis of new model-theoretical results on the structure of models in our case represent an important role for the development of a general theory of models. As was noted earlier, in this work, we study the Jonsson sets, which are subsets of the semantic model of the considered Jonsson theory. Next, on the special closures of these subsets we will consider some inductive theory. In fact, at some closure, the Jonsson theory will be considered.

At the end of the 80s, Kazakh mathematician T.G. Mustafin turned to this theme and defines the main aims and methods of work with Jonsson theories. He obtained results related both to arbitrary Jonsson theories and to specific examples of Jonsson theories.

As was noted, the concept of a companion was first introduced by A. Robinson. Two of his important contributions to this area were the concepts of model completeness and model completion. These ideas eventually led to the study of model companions. The study of model companions in the works of B. Poise, J. Barweis, P. Eklof and D. Sabbach.

Further, the study of Jonsson theories regarding the various model-theoretical properties of their companions, including J-stability, was continued by A.R. Yeshkeyev.

The definition of new classes of positive Jonsson theories is an important step in the development of Jonsson theories. Positive Jonsson analogues of the work of F. Weispfenning were obtained for the positive lattice of existential formulas of the considered theory . The concept of positive Jonsson theories was introduced after the appearance of a series of works by I. Ben-Yaacov, because

both concepts of the positivity of theory coincide with each other for a minimal fragment of the considered theory.

There is a class of mathematical structures, as metric spaces, which is not a Jonsson class, but is positively Jonsson in the sense of I. Ben-Yaacov and, in particular, in the sense of A.R. Yeshkeyev for a minimal fragment. Therefore, the concept of positivity in the sense of A.R. Yeshkeyev is nontrivial. It should be noted that there are various regular ways of moving from an arbitrary theory to a Jonsson theory, which preserves the original class of existentially closed models. One of these methods is the morlization of theory. Thus, the study of model-theoretical properties of Jonsson theories is actual task.

The aim of this work is the study of the properties of companions of fragments of Jonsson sets. In particular, to study Jonsson subsets of the semantic model of a fixed Jonsson theory, which discusses the similarity of Jonsson sets, the properties of lattices of existential formulas of Jonsson fragments, also the cosemantic property for positive fragments and their models in an enriched signature. In addition, to study strongly minimal sets on the pregeometry of the semantic model of a fixed Jonsson theory, model-theoretic properties of $\#_-$ companions of Jonsson sets, ω- stability of $\#_-$ companion of the theory of Jonsson pairs in the theory of Abelian groups, and an uncountable central type of Robinson spectrum.

The objects of research are Jonsson theories and their classes of models. In particular, companions of fragments of Jonsson sets are considered in work.

Research methods. In the framework of scientific research, the general methods of the classical model theory related to the study of complete theories, as well as the methods of universal algebra, were applied. And also the semantic method was used, the essence of which is to transfer of the elementary properties of a center of Jonsson theory to this theory itself. In the case when direct translation from the Jonsson theory to its center is possible, as a rule, we work, even in the imperfect case, only with the class of existentially closed models.

The novelty of the dissertation research. All concepts associated with the study of Jonsson sets and their fragments are new and introduced to study various new subclasses of Jonsson theories.

The objectives of the study. The Jonsson theories, generally speaking, are incomplete, and the apparatus for studysuch theories is insufficient for the modern realities of model theory. Therefore, the first stage of the study is to redefine the main results of the classical model theory. This work is related to the concept of a fragment of Jonsson set. Due to the incompleteness of Jonsson theories, we try to work with the perfect case, since in this case, when studying the properties of the first order theory itself, we use the semantic method. This method works well for all types of studies. But since methods for only complete theories have been developed for such studies, in this work we consider analogues of the results for Jonsson theories in the language of Jonsson sets

Theoretical and practical sifnificance of the research. The work is theoretical in nature. The study of fragments of Jonsson sets and the model-theoretical attributes associated with them can be used in further studies of

model-theoretical properties of Jonsson theories and their classes of models in classical model theory.

The main provisions for the defense of the dissrrtation.

1) The syntactic similarity of Jonsson sets is obtained.

2) The criterion of cosemanticness for positive fragments and their models in an enriched signature is obtained.

3) Categoricity of # - the companion of a fragment of Jonsson subset of the semantic model of a fixed Jonsson theory

4) The Stone Algebra of existential formulas of the # -companion of a fragment of a Jonsson subset of the semantic model of a fixed Jonsson theory is described

5) A criterion of ω - stability of #- a companion of the theory of Jonsson pairs of the theory of abelian groups

6) A criterion of uncountable categoricity is obtained for the class of the Robinson spectrum in the language of central types.

Publications and personal contribution of the author. The main results of the dissertation were published in 10 works, of which 1 article was published in a journal included in the Scopus database, 3 articles were published in journals recommended by the Committee for Control in Education and Science of the Ministry of Education of the Republic of Kazakhstan, 5 works were published in materials of international scientific conferences 1 article published in a periodical of a far abroad country (Czech Republic)

The structure and volume of the dissertation. The volume of the dissertation is 105 pages. The work consists of the following structural elements: notation and abbreviation, introduction, the main three sections, conclusion and references, containing 85 items.

Main content of dissertation work

The work consists of 3 sections. Section 1 gives general information on the theory of companions, the Jonsson theory and its companions, and the class of existentially closed models of a fixed Jonsson theory.

Subsection 1.1 gives the basic concepts and results of the classical model theory concerning companions.

Subsection 1.2 is an overview, it summarizes the basic information about Jonsson theories and companions of Jonsson theory and their relationship to each other.

Let T be a Jonsson theory. A companion of Jonsson theory T is called a theory $T^{\#}$ of the same signature that satisfies the following conditions:

1) $\left(T^{\#}\right)_{\forall} = T_{\forall}$;

2) for any Jonsson theory T', if $T_{\forall} = T'_{\forall}$, then $T^{\#} = \left(T'\right)^{\#}$.

3). $T_{\forall\exists} \subseteq T^{\#}$

The natural interpretations of the companion $T^{\#}$ are $T^{*}, T^{f}, T^{M}, T^{e}$, where

T^* is the center of Jonsson theory T , T^f is the forcing companion of Jonsson theory T , T^M is the model companion of t theory T , $T^e = Th(E_T)$,where E_T is the class of existentially closed theory models of T .

Theorem 1.2.16 Let T be Jonsson theory. Then the following conditions are equivalent:

1) T is perfect;

2) T has a model companion

Theorem 1.2.17 Let T be Jonsson theory. Then the following conditions are equivalent:

1) T is perfect;

2) T^* is a Jonsson theory;

3) T^f is a Jonsson theory;

4) T^e is a Jonsson theory; where $T^e = Th(E_T)$;

5) T^e is a complete theory ;

6) T^0 is a complete theory ;

Lemma 1.2.6 If $T^\#$ is the companion of the Jonsson theory T and T^M is the model companion of T , then $T^\# = T^M$.

Lemma 1.2.7. Let T_1 and T_2 be Jonsson theories. Then the following conditions are equivalent:

1) T_1 and T_2 are mutually model cosistent;

2) $T_1^\# = T_2^\#$.

Lemma 1.2.9. Let T be Jonsson theory. Then the following conditions are equivalent:

1) T is perfect;

2) T is super - Jonsson;

3) T^f is -Jonsson;

4) T is *-Jonsson;

5) T is ^{e}-Jonsson

Subection 1.3 is devoted to the description of existentially closed models in the framework of the study of fixed Jonsson theory.

In this section we consider some properties of existentially closed models in the class of Jonsson theories, i.e. any considered theory is a Jonsson theory.

Theorem 1.3.12 For any countable elementary submodel of a semantic model A of a perfect Jonsson theory T , the following conditions are equivalent:

1) A is symmetric.

2) A is existentially closed

The existence of a model companion of T is closely related to T - existentially closed models.

Theorem 1.3.18 Let T a perfect Jonsson theory. $T^\#$ is a model companion of T . For any model A of T , the following conditions are equivalent:

1) A is a model $T^\#$ where $T^\#$ is #- companion of Jonsson theory

2) A is T -existentially closed.

3) A is T - uniformly existentially closed

Theorem 1.3.20. Let T be a perfect Jonsson theory. The following conditions are equivalent:

1) T has a model companion.

2) T has a saturated model which is T - existentially closed.

3) T has a * -saturated model which is T - existentially closed.

4) T has a model which is T - uniformly existentially closed.

Section 2 describes the model-theoretic properties of fragments of definable closures. Subsection 2.1 describes the Jonsson subsets of the semantic model of the fixed Johnson theory.

Definition 2.1.12 A set X is called Jonsson in theory T if it satisfies the following properties:

a) X is Σ - definable subset of C, where C is a semantic model of the theory T;

b) $dcl(X)$ is a carrier of some existentially closed submodel C, where $dcl(X)$ is a definable closure of the set X.

Using the definitions of Jonsson sets, it is possible to transfer properties for Jonsson theories to arbitrary subsets of the semantic model.

We say that two Jonsson sets (equivalent, cosemantic, categorical) if, accordingly, will be (Jonsson equivalent, cosemantic, categorical, syntactically similar, semantically similar, etc.) models that are obtained with the corresponding closure of these sets.

Two (algebraic) Jonsson sets are syntactically similar to each other if elementary theories of their respective closures are syntactically similar.

Let T be an arbitrary Jonsson theory, thenv $E(T) = \bigcup\limits_{n < \omega} E_n(T)$, where $E_n(T)$ is the lattice of existential formulas with exactly n free variables.

Definition 2.1.14 Let T_1 and T_2 be Jonsson theories. We shall say that, T_1 and T_2 is J -syntactically similar, if and only if there exist a bijection $f : E(T_1) \rightarrow E(T_2)$ such that

1) the restriction of f up $E_n(T_1)$ is isomorphism of the $E_n(T_1)$ and $E_n(T_2)$, $n < \omega$;

2) $f(\exists v_{n+1}\varphi) = \exists v_{n+1} f(\varphi)$, $\varphi \in E_n(T)$, $n < \omega$,

3) $f(v_1 = v_2) = (v_1 = v_2)$.

Definition 2.1.19 Algebraically Jonsson sets X and Y are syntactically similar to each other if syntactically similar, respectively, its T_{M_1} and T_{M_2} where acl $(X) = M_1$, acl $(Y) = M_2$.

If X and Y are syntactically similar to each other, then we have the following result:

Theorem 2.1.1 Let T_{M_1} and T_{M_2} be $\exists$-complete perfect Jonsson theories.

Then the following conditions equivalent:

1) $T^*_{M_1}$ and $T^*_{M_2}$ are syntactically similar as complete theories as in [40];

2) T_{M_1} and T_{M_2} are J -syntactically similar.

Also in this subsection fragments of Jonsson sets are considered, which are subsets of the semantic model of a certain Jonsson theory of countable first-order language. The results are obtained that establish the relationship between the properties of the fragment and the Jonsson theory, the central completion of this Jonsson theory, and the properties of the lattice of equivalence classes of existential formulas with respect to the fragment under consideration. In terms of the lattice of formulas, conditions are found for eliminating the quantifiers of the central complement of the Jonsson theory, positive model completeness of the central complement of the Jonsson theory, the perfectness of the Jonsson theory, and the Jonsonness of the central complement of the fragment. Let T be Jonsson theory complete for existential sentences, C its semantic model.

Subsection 2.2 describes various methods for classifying fragments of Jonsson sets with respect to the cosemanticness of the closures of these sets in existentially simple convex Jonsson theories. Also the syntactic and semantic properties of their models through the concept of cosemantics for fragments of the considered Jonsson sets are considered

At the same time, through the concept of cosemantics, the syntactic and semantic properties of their models were considered

Let T be the original Jonsson theory in language L. Let X_1, X_2 be Jonsson sets in theory T. Then let $M_{X_1} = dcl(X_1)$ be a closure of the Jonsson set X_1, $M_{X_2} = dcl(X_2)$ be a closure of the Jonsson set X_2. Then $T_{M_{X_2}}$ is a fragment of X_1, $T_{M_{X_2}}$ is a fragment of X_2 .

Definition 2.2.3 Models A and B of signature σ are called cosemantic (symbolically $A \bowtie_J B$) if, for any Jonsson theory T_1 such that $A \vDash T_1$, there is a Jonson theory T_2 cosemantic with T_1 such that $B \vDash T_1$. And vice versa

The following results are obtained.

Theorem 2.2.1 Let $T_{M_{X_1}}$ and $T_{M_{X_2}}$ be fragments, respectively, of Jonsson sets X_1 and X_2 in an existentially simple convex Jonsson theory. Moreover, C_1 is the semantic model of $T_{M_{X_1}}$, C_2 is the semantic model of $T_{M_{X_2}} T_{M_{X_2}}$. Then the following conditions are equivalent

1) $C_1 \bowtie_J C_2$,

2) $C_1 \equiv_J C_2$,

3) $C_1 = C_2$.

Theorem 2.2.2 Let M_{X_1}, M_{X_2} be existentially closed submodels of the semantic model of a perfect Jonsson theory, X_1, X_2 be Jonsson sets in this theory.

Then the following conditions are equivalent:

1) $M_{X_1} \rhd\lhd_j M_{X_2}$,

2) $\forall\exists(M_{X_1}) \rhd\lhd_j \forall\exists(M_{X_2})$.

And also in this subsection, the property of cosemanticness for positive fragments and their models in an enriched signature was considered.

Let T be an arbitrary $\Delta - PJ$ - Jonsson fragment in the first-order language of the signature σ. Let C be a semantic model of the Jonsson fragment T. $A \subseteq C$

Let

$$\sigma_\Gamma(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma,$$

$T_\Gamma^{PJ}(A) = Th_{\forall\exists^+}(C, a)_{a \in A} \cup \{P(c_a) \mid a \in A\} \cup \{P(c)\} \cup \{"P \subseteq"\}$ where $\{"P \subseteq"\}$ is an infinite set of sentences expressing the fact that the interpretation of the symbol P is an existentially closed submodel in the signature language σ.

Consider all the completions of the Jonsson fragment T^* for the Jonsson fragment T in the signature language σ_Γ, where $\Gamma = \{c\}$. Since T^* is $\Delta - PJ$ - Jonsson fragment, it has a center and we denote it by T^c. By restriction the theory T^c to a signature σ, the theory T^c becomes a complete type. This type is called the central type of the Jonsson fragment T.

Definition 2.2. 4 Let A be some infinite signature model σ. A is called $\Delta - PJ$ − model, if the set of sentences $T_\Gamma^{PJ}(A)$ is $\Delta - PJ$ - Jonsson fragment in an enriched language.

The Jonsson fragment $T_\Gamma^{PJ}(A)$ will be denoted by $\forall\exists^+(A)$.

Theorem 2.2.4 Let A and B be $\Delta - PJ$ models of signature $\sigma_\Gamma(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma$. Then the following conditions are equivalent::

1) $A \rhd\lhd_{PJ}^\Delta B$,

2) $\forall\exists^+(A) \rhd\lhd_{PJ}^\Delta \forall\exists^+(B)$.

Theorem 2.2.5. Let T_1^* and T_2^* be $\Delta - PM$ - Jonsson fragments, C_1 is semantic model of T_1, C_2 is semantic model of T_2. Then the following conditions are equivalent

1) $C_1 \rhd\lhd_{PM}^\Delta C_2$,

2) $C_1 \equiv_{PM}^\Delta C_2$,

3) $C_1 = C_2$

Subsection 2.3 is devoted to the description of strongly minimal sets on the pregeometry of a semantic model of a fixed Jonsson theory
In this subsection, we redefine the basic concepts on the formul subsets of some extentional-closed model for some fixed Jonsson theory. With the help of new concepts in the frame of Jonssoness features, pregeometry is given on all subsets of Jonsson theory's semantic model. Minimal structures and, correspondingly, pregeometry and geometry of minimal structures are determined. We consider the

concepts of dimension, independence, and basis in the Jonsson strongly minimal structures for Jonsson theories.

Definition 2.3.10 . Let X be subset of semantic model of fixed Jonsson theory and let cl $:\mathcal{P}(X) \rightarrow \mathcal{P}(X)$ be an operator on the power set of X. We say that (X, cl) is a Jonsson pregeometry if the following conditions are satisfied.

   a)   If $A \subseteq X$, then $A \subseteq cl(A)$ and $cl(cl(A)) = cl(A)$.

   b)   If $A \subseteq B \subseteq X$, then $cl(A) \subseteq cl(B)$.

   c)   (exchange) $A \subseteq X$, $a, b \in X$,   and $a \in cl(A \cup \{b\})$,   then
        $a \in cl(A), b \in cl(A \cup \{a\})$.

   d)   (finite character) If $A \subseteq X$ and $a \in cl(A)$, then there is a finite $A_0 \subseteq A$
        such that $a \in cl(A_0)$.

        We say that $A \subseteq X$ is closed if $cl(A) = A$.

   Theorem 2.3.2 Let (X, cl) be J- pregeometry. The following are equivalent.

   1) (X, cl) is modular.

   2)   If $A \subseteq X$ is closed and nonempty, $b \in X$, and $x \in cl(A, b)$, then there is $a \in A$ such that $x \in cl(a, b)$

   3)   If $A, B \subseteq X$ are closed and nonempty, and $x \in cl(A, B)$, then there are $a \in A$ and $b \in B$ such that $x \in cl(a, b)$

   Subsection 3.1 considers the countable and uncountable categoricity of # -companion of a perfect fragment of a certain Jonsson subset of a semantic model of a fixed Jonsson theory.

   In this section, we consider the properties of $Fr^{\#}(A)$ - # -companion of fragment of Jonsson set of a perfect fixed Jonsson theory T.

   Theorem 3.1.7   Let $Fr(A)$ be $\forall \exists$ - complete perfect strongly convex Jonsson fragment of Jonsson set A. Then the following conditions are equivalent:

   1) $Fr^{\#}(A)$ is ω-categorical;

   2) $Fr(A)$ is ω-categorical.

   Theorem 3.1.10. Let $Fr(A)$ be Jonsson fragment, which is an existentially prime perfect Jonsson theory complete for an existential sentences of Jonsson universal theory, for which R_1 is satisfied. Then the following are equivalent:

   1) the theory $Fr^{\#}(A)$ is ω_1-categorical,

   2) any countable model in $E_{Fr(A)}$ has an algebraically prime model extension in $E_{Fr(A)}$.

   Subsection 3.2 is devoted to the description of the Stone algebra of existential formulas of the # -companion of fragment of Jonsson subset of the semantic model of a fixed Jonsson theory

   Let T be Jonsson theory of a countable language L, A be Jonsson subset of the semantic model of T, $Fr(A)$ is a fragment of Jonsson set A. Let $E_n(Fr(A))$ be the   distributive   lattice   of   equivalence   classes   of

$$\varphi^{Fr(A)} = \{ \psi \in E_n(L) | Fr(A) |- \varphi \leftrightarrow \psi \}, \ \varphi \in E_n(L), E(Fr(A)) = \bigcup_{n<\omega} E_n(Fr(A))$$

Theorem 3.2.1 Let $Fr(A)$ be the perfect fragment of Jonsson set A, $Fr^{\#}(A)$ is $\#$-companion. Then

1) $Fr^{\#}(A)$ admits elimination of quantifiers if and only if every $\varphi \in E_n(Fr(A))$ has quantifier-free complement;

2) $Fr^{\#}(A)$ is positively model-complete if and only if every $\varphi \in E_n(Fr(X))$ has an existential complement.

Theorem 3.2.2 Let $Fr(X)$ be a perfect fragment of Jonsson set A. $Fr^{\#}(A)$ is $\#$-companion. Then the following conditions are equivalent:

1) $Fr(X)$ is perfect;

2) $E_n(Fr(X))$ is weakly complemented;

3) $\varphi \in E_n(Fr(X))$ is a Stone algebra.

Theorem 3.2.3 Let $Fr(X)$ be a perfect fragment of Jonsson set A. $Fr^{\#}(A)$ is $\#$-companion. Then the following conditions are equivalent:

1) $Fr^{\#}(A)$ is Jonsson theory;

2) every $\varphi^{Fr(A)} \in E_n(Fr(A))$ has a weak quantifier-free complement.

Subsection 3.3 is devoted to the study of model-theoretical questions of Abelian groups in the framework of the study of Jonsson theories, in particular, a criterion for ω-stability of $\#$-companion theory of Jonsson pairs of the theory of Abelian groups was obtained.

We consider the language L_P obtained by adding the one-place predicate $P(x)$ to the language L. Denote by T_P the theory obtained by adding to T axioms, which state that the interpretation of P is also a model of the theory T.

The model of theory T_P is called the Jonsson pair (J-pair) of models T. We will denote this pair (N, M), where M is the interpretation of the predicate $P(\bar{x})$ In this pair, we call N - a large model, and M is a small model
We denote by $T_{P_{AG}}$ the theory of Jonsson pairs of the theory of Abelian groups.

Proposition 3.3.1 The theory $T_{P_{AG}}$ is the perfect Jonsson theory.

Theorem 3.3.1 Let $T_{P_{AG}}$ be the theory of Jonsson pairs of the theory of Abelian groups, then the following conditions are equivalent:

1) $T_{P_{AG}}$ is J - ω - stable;

2) $T_{P_{AG}}^{*}$ - ω -stable;

3) $T_{P_{AG}}$ has JSB property

In subsection 3.4, model-theoretical properties of the Robinson spectrum of an arbitrary model of an arbitrary signature are studied. A criterion of uncountable categoricity is obtained for the considered class of Robinson spectrum in the language of central types.

Definition 3.4.4 A theory T is called Robinson if it satisfies the following conditions:

  1) T has at least one infinite model;

  2) T universally axiomatizable;

  3) T allows the property of joint investment;

  4) T allows amalgam property.

  Let T be Robinson theory, A be an arbitrary model of signature σ. The Robinson spectrum of a model A is the set:

  $RSp(\mathsf{A}) = \{T \mid T$ is Robinson theory in language σ and $\mathsf{A} \in \mathrm{Mod}(T)\}$.

  The cosemantic relation on the set of theories is an equivalence relation. Then $RSp(\mathsf{A})/_{\bowtie}$ is the factor set of the Robinson spectrum of the model A with respect to $\bowtie$.

  Using the scheme of obtaining the central type for the class, we obtain the central type of the class. Consider the class $[T] \in JSp(\mathsf{A})/_{\bowtie}$. Let be $Th(C_{[T]}) = [T]^{*}$. For each $\Delta \in [T] \in JSp(\mathsf{A})/_{\bowtie}$ we denote the theory obtained by the scheme (#) by $\overline{\Delta}$. Consider a class $[\overline{T}]$ and then the class $[\overline{T}]^{*}$, after restriction by the scheme (#), becomes the central type of the class $[\overline{T}]$ and is denoted by $P^{c}_{[\overline{T}]}$.

  Theorem 3.4.2. Let $[T]$ be a hereditary class from $RSp(\mathsf{A})/_{\bowtie}$, then the following conditions are equivalent

  1) Any countable model from $E_{[\overline{T}]}$ has an algebraically simple model extension in $E_{[\overline{T}]}$

  2) $P^{c}_{[\overline{T}]}$ is strongly minimal type, where $P^{c}_{[\overline{T}]}$ is the central type of $[\overline{T}]$

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