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Report on Thesis by D.R. Beisenova

This report is based on her doctoral thesis "The separability and spectral properties of high order differential and difference operators".

The dissertation is devoted to the questions of the correctness and separability of high-order singular difference and differential operators having unbounded intermediate coefficients. Hereinafter, by a singular difference operator we mean an operator corresponding to a system of an infinite number of linear difference equations.

The second and higher order singular difference and differential operators are used in quantum mechanics, stochastic analysis, and stochastic differential equations. They have been studied intensively since the 20-30s of the last century in the works of Weil, Schrödinger, Smoluchowski, Fokker, Planck, Ornstein, Uhlenbek, Kolmogorov and many others. The results of the research can be found, for example, in the famous monographs of Naimark, Fedoryuk, Reed and Simon, Agarwal, Cuevas and Lizama.

However, in these studies, it is assumed that the growth at infinity of the modules of the intermediate coefficients of the operator and their derivatives are limited to some degree of the lowest coefficient. Such a strict condition severely limits the applicability of the above operators when modeling specific real pro-

cesses. In the works of S. Fornaro and L. Lorenzi (2007), M. Hieber and O. Sawada (2005), G. Metafune, D. Pallara and V. Vespri (2005), M. Hieber, L. Lorenzi, J. Pruss, A. Rhandi and R. Schnaubelt (2009) a second-order singular differential operator whose intermediate coefficient does not obey to the potential, but it grows no faster than $|x| \ln |x|$ ($|x| \gg 1$) is studied. Recently, the works of K.N. Ospanov, A. Zulkhazhbay and R.D. Akhmetkaliyeva (2012, 2016) the correctness of the second-order differential and difference operators, whose intermediate coefficients grow faster than $|x| \ln |x|$ ($|x| \gg 1$) is established.

The thesis of D. Beisenova is a logical continuation of the above studies. In this thesis the questions of correctness and separability of high-order difference and differential operators with rapidly growing and oscillating intermediate coefficients, as well as the structure of their spectrum were considered.

And the following results were obtained:

A Hardy type difference weight inequality is proved, which gives an estimate of the weighted norm of a numerical sequence in terms of the weighted norm of its higher order differences. This inequality is used to establish the invertibility of high-order degenerate differential operators. Further, the conditions of bounded invertibility of a high even order singular difference operator with dominant intermediate coefficients in a Hilbert space are obtained, a complete description of its domain and a coercive estimate of the elements from the domain through the norm of the operator itself is given.


For a second order degenerate difference operator with strongly oscillating intermediate coefficients, the sufficient conditions for correctness and coercivity are found, and a theorem of Molchanov type on the discreteness of its spectrum is proved. The separability of a second-order differential operator with complex-valued intermediate coefficients is proved with a corresponding coercive estimate of the elements of its domain. Sufficient conditions for the bounded invertibility of a fourth-order differential operator with a dominant and rapidly oscillating intermediate coefficient are obtained, and a separability estimate for elements of

its domain is established.

In my opinion, the novelty of the above results is as follows: a) the separability of a even order difference operator is established, b) in contrast to the case of second-order differential operators, a coercive estimate is obtained without the traditional condition for the oscillation of the intermediate coefficients of the operator, c) a criterion for the discreteness of the spectrum of a degenerate difference operator is found when the order of the operator is two, d) a coercive estimate is obtained for a fourth-order differential operator when it is degenerate, and its intermediate coefficient fluctuates rapidly.

All the main conclusions of the thesis are formulated in the form of different theorems, which are equipped with detailed evidence. All scientific results are reasonable and reliable. Auxiliary statements for studying problematic issues in each section are presented in the form of lemmas and remarks, all of them are rigorously proved. This shows that D.R. Beisenova quite deeply understands general scientific methods, as well as the logic and technology of scientific research.

As a conclusion, D.R. Beisenova obtained new and deep scientific results for singular difference and differential operators with dominant intermediate coefficients. Therefore, I believe that the doctoral dissertation "The separability and spectral properties of high order differential and difference operators" satisfies all the requirements for dissertation for the degree of Philosophy Doctor (PhD). I recommend to accept the D.R. Beisenova's work as a PhD dissertation in the specialty 6D060100 - MATHEMATICS.

Sincerely Yours,

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