

URKEN GULZHAN ATKENKYZY
SIMILARITY OF JONSSON THEORIES
ANNOTATION

dissertation for the degree of Doctor of Philosophy (PhD)

specialty 6D060100 – Mathematics

The relevance of the topic. Model Theory is one of the most important branches of modern mathematics. Model Theory underlies mathematical logic, universal algebra, abstract algebra, topology, and geometry. One of the famous mathematicians J. Keisler considered the theory of models, dividing it into two directions: “Eastern” and “Western”. These names are conditional because they are associated with the place of residence of the founders of model theory. A. Robinson and A. Tarski lived on the east and west coasts of the United States, respectively. Western Model Theory studied complete theories, while the Eastern direction studied incomplete theories.

The problem of classifying complete theories in Model Theory posed in the dissertation work is currently the most relevant problem in this area. The Jonsson theories being studied are, generally speaking, incomplete, so the need to classify these theories according to the principle of similarity seems obvious. In fact, the concept of similarity is a concept introduced for complete theories. The study of this concept is an important new topic for Jonsson theories. In addition, research methods are new. The syntactic and semantic similarity of Jonsson theories, the syntactic similarity of a polygon, and the similarity of the Jonsson spectrum class are investigated in the dissertation

The relevance of studying the similarity of Jonsson's theories, including syntactic and semantic similarity, through model-theoretical properties is not in doubt, and it is clear that it is an interesting and complex problem of the problems of the models' theory. More precisely, this work examines results related to the syntactic and semantic similarity of Jonsson theories. A new and relevant method for studying Jonsson theories is to study these theories using the concepts of syntactic and semantic similarity of complete theories since the center of Jonsson theory is a complete theory.

At the same time, the concepts of syntactic and semantic similarity, as it turns out, retain all the important model-theoretic properties, such as stability, categoricity, and Morley rank.

The goal of the work. The main goal of the dissertation research is to study the syntactic and semantic similarity of Jonsson theories.

Research objectives: The content of the dissertation is a study of the following problems:

1. To show that the theories of Boolean rings and Boolean algebra are syntactically similar and mutually interpretable;
2. To consider whether the theory of Boolean algebra interprets the theory of Abelian groups;
3. To find for any \exists - complete perfect Jonsson theory some syntactically similar \exists - complete perfect Jonsson theory of polygons;
4. To obtain a description of the syntactic and semantic similarity of perfect fragments of the Jonsson subset of the semantic model of the existential simple convex Jonsson theory;
5. To obtain results within the framework of a study of the similarity of classes of Jonsson spectrums;
6. Obtain results describing the closure operator using the concept of a closure operator that defines the original geometry within the framework of the study of non-forkable relations on subsets of the semantic model of some Jonsson theory, if these subsets are Jonsson.

The object of the research: The object of research is the similarity of Jonsson theories.

The subject of the research: The subject of research are Jonsson theories, as well as the determination of the basic properties of syntactic and semantic similarity of Jonsson theories.

Research methodology. One of the main methods of the theory of classical models in the dissertation work is the semantic approach. The main content of this approach is to transfer the properties of the considered theory's center to the theory itself.

Scientific novelty. A study of the basic concepts of Jonsson theories and the connection between syntactic and semantic similarity from the point of view of model-theoretic properties.

Theoretical and practical value of the work.

The results obtained in the dissertation are theoretical and can be used in further research within the framework of Model Theory and in the study of semantic and syntactic similarity of Jonsson theories. Since the main questions related to this topic belong to the classical form of Model Theory, they are used in applied and scientific areas of mathematics.

Provisions submitted for presentation.

The following are submitted for presentation:

1. It is shown that the Boolean ring and Boolean algebra are syntactically similar and mutually interpretable theories;
2. It is shown that the theory of Boolean algebra interprets the theory of Abelian groups;
3. For any \exists -complete perfect Jonsson theory, some syntactically similar \exists -complete perfect Jonsson theory of polygons is found;

4. A description of the syntactic and semantic similarity of perfect fragments of the Jonsson subset of the semantic model of the existential simple convex Jonsson theory is obtained;

5. Results were obtained as part of a study of the similarity of classes of Jonsson spectrums;

6. Within the framework of the study of the non-forking relation on subsets of the semantic model of some Jonsson theory, if these subsets are Jonsson, a description of the closure operator is obtained using the concept of a closure operator that defines the initial geometry in these subsets.

The credibility and validity are ensured by the constructiveness of the used methods. General conclusions are presented in the form of theorems and their proofs are presented.

Approbation of the work.

The main results of the dissertation were confirmed and discussed at the following conferences and seminars:

1. 6th World Congress and School on Universal Logic (Vichy (France), 2018 – June 16-26).

2. 16th Asian Logic Conference (Kazakhstan, Nur-Sultan, Nazarbayev University, 2019 – June 17-21);

3. Traditional international April mathematical conference (Kazakhstan, Almaty: Institute of Mathematics and Mathematical Modeling, 2019 – April 3-5);

4. International conference “Current problems of analysis, differential equations and algebra” (EMJ-2019) (Kazakhstan, Nur-Sultan: Eurasian National University named after L.N. Gumilyov, 2019 – October 16-19);

5. International scientific conference “Theoretical and applied problems of mathematics, mechanics and computer science” (Kazakhstan, Karaganda: Karaganda University named after Academician E.A. Buketova, 2019 – June 12-13).

6. Traditional international April mathematical conference (Kazakhstan, Almaty: Institute of Mathematics and Mathematical Modeling, 2024 – April 16-19 and 22);

7. Seminar of applied mathematics of Karaganda University named after academician E.A. Buketov (laboratory “Mathematical Logic”).

Publications. The main results of the dissertation were published in 13 scientific papers: 1 article in the journal «Bulletin of the Karaganda University. Mathematics Series», indexed in the Scopus database, percentile – 35), 4 articles in journals recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan and 7 works published in materials of international scientific conferences, 1 article published in Republican scientific conference.

In the works performed with co-authors, the contribution of each of the coauthors is equal.

The structure and scope of the dissertation.

The 84-page dissertation consists of the following structural elements: introduction, three chapters, conclusion, and list of the used sources. The chapters of the dissertation are closely related to each other.

The first chapter of the dissertation examines the basic concepts of model theory, in particular, such basic topics as signatures of algebraic models, first-order language, monomorphisms, elementary equivalence, and model completeness are considered.

The second chapter provides the necessary information on the Model Theory studied within the framework of Jonsson theories. In this chapter, as part of the study of Jonsson theories, generally speaking, incomplete theories, companions, and classical problems of Model Theory, such as syntactic and semantic similarity of complete theories, were considered.

The third chapter of the dissertation shows the main results, in particular, the similarity of classes of Jonsson spectrum is considered. Each paragraph respectively presents the admissibility of Jonsson theories, the connection between the semantic and syntactic similarity of Jonsson theories, the syntactic similarity of some Jonsson theories and their connection with admissibility, the main conclusions and evidence of the similarity of classes of Jonsson spectrums.

The number of sources used is 56.

Keywords. Jonsson theory, Jonsson perfect theory, semantic model, syntactic and semantic similarity, Jonsson spectrum, cosemanticity, admissibility, interpretation, S-polygon.