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ON ATTRACTORS TO THE STOKES SYSTEM OF EQUATIONS IN PLANAR POROUS MEDIUM

ABSTRACT
of thesis for the degree of Doctor of Philosophy (PhD)
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The relevance of the topic. Modern materials science and applied problems of modern physics, biology and chemistry lead, in particular, to the study of processes in micro-homogeneous media (skeletons, porous media, composite materials, nanostructures, etc.). Such problems are difficult to solve using numerical methods and computational tools, since they require the study and solution of systems of algebraic equations with billions of unknowns and equations. In this case, the methods of asymptotic analysis and averaging theory come to the rescue, which allow us to write out significantly simpler problems in areas without micro-heterogeneities, the solutions of which are close to the solutions of the original problems. In this paper, we study the asymptotic behavior of the attractors of the initial boundary value problem for a two-dimensional system of Navier–Stokes equations in an anisotropic medium with locally periodic fine-grained obstacles whose sizes depend on the small parameter , with $\varepsilon \to 0$ this parameter tending to zero. Problems in perforated areas (in areas with fine-grained obstacles) attract a lot of attention from mathematicians (see, for example,[1]–[6]) .

In this paper, we study the asymptotic behavior of attractors of the initial boundary value problem for a two-dimensional system of Navier-Stokes equations in an anisotropic medium with periodic and locally periodic fine-grained obstacles, the sizes of which depend on the small parameter ε, with this parameter tending to zero. Problems in perforated areas (in areas with fine–grained obstacles) attract great attention from mathematical specialists (see, for example, [7-13]).

Some results on averaging for various problems in perforated areas can be read in monographs [14-20], as well as read a detailed bibliography there.

Attractors describe the behavior of solutions of dissipative nonlinear evolutionary equations at long times, and also characterize the stability and instability of the limiting structures of the corresponding dynamical systems (see, for example, monographs [20 p. 467-486; 21-33] and references therein). The problems for autonomous and non-autonomous two-dimensional Navier-Stokes equations with oscillating terms were studied in [34-36].

We note some results on the averaging of attractors that have appeared recently (see [38]). In [38, pp. 95-103; 39-44] and [44, pp. 289-308; 45, 46], the averaging of attractors of scalar evolutionary reaction-diffusion equations with dissipation in periodic perforated regions was studied.

In the dissertation work, we are interested in the asymptotic behavior of the trajectory attractors of the Navier-Stokes system of equations and the general system

of Navier-Stokes equations with rapidly oscillating terms in a perforated region (an area with obstacles). We study the weak convergence and limiting behavior of attractors when the small parameter ε tends to zero (here the small parameter characterizes the diameter of cavities (obstacles) and the distance between them in the medium).We have shown that the attractors of the initial initial boundary value problems converge to the attractors of the limiting (averaged) initial boundary value problems for systems of Navier-Stokes equations with modified main parts and with additional potentials (see similar problems in [44, p. 289-308], [20, p.467-486], [21, p. 655-683]).

The purpose of the work. The purpose of this work is to study the behavior of attractors of a two-dimensional system of Navier-Stokes equations and a generalized two-dimensional system of Navier-Stokes equations given in areas with small holes, with a small parameter characterizing the size of these holes and the distance between them to zero.

Research objectives.

- To study the limiting behavior of trajectory attractors of the initial boundary value problem for a two-dimensional system of Navier-Stokes equations in a periodic porous medium;
- To study the limiting behavior of trajectory attractors of the initial boundary value problem for a two-dimensional system of Navier-Stokes equations in a locally periodic porous medium;
- To study the limiting behavior of trajectory attractors of the initial boundary value problem for a generalized two-dimensional system of Navier-Stokes equations in a locally periodic porous medium.

The objects of research. Trajectory attractors of a two-dimensional system of Navier-Stokes equations in regions with periodic and locally periodic small obstacles.

Research methods. Methods of asymptotic analysis and the theory of averaging initial boundary value problems of partial differential equations are used to study the tasks set.

Main provision. The following new scientific results were obtained in the work:

1. The convergence condition is obtained and the limiting behavior of the trajectory attractors of the plane Navier-Stokes problem in a periodic porous medium is described;
2. The convergence condition is obtained and the limiting behavior of the trajectory attractors of the two-dimensional Navier-Stokes problem in a locally periodic perforated medium is described;
3. The convergence condition is obtained and the limiting behavior of the trajectory attractors of the two-dimensional Navier-Stokes problem for an anisotropic fluid with variable viscosity in a locally periodic perforated medium is described.

Description of the main results of the study

In the first part of the dissertation, the basic concepts of trajectory attractors of non-autonomous evolutionary equations are given and general theorems are formulated.

In the second part of the research work, the conditions for convergence of the trajectory attractors of the plane Navier-Stokes problem in a periodic porous medium are obtained.

In the third section, the conditions for convergence of the trajectory attractors of the two-dimensional Navier-Stokes problem in a locally periodic perforated medium are obtained.

In the fourth section, conditions for convergence of trajectory attractors of the two-dimensional Navier-Stokes problem for an anisotropic fluid with variable viscosity in a locally periodic perforated medium are obtained.

Substantiation of the novelty and importance of the results obtained.

The scientific results obtained in the work are new and have a theoretical character. They describe the long-term behavior of solutions to a two-dimensional system of Navier-Stokes equations in a periodic and locally periodic perforated medium. These results can also be used in applied mathematics for numerical simulation of fluid motion in flat areas with small obstacles.

The obtained scientific results can be used as sections of elective courses on partial differential equations in the preparation of scientific personnel in graduate and doctoral studies.

Approbation of the work. The main results of the work were presented at the following conferences:

1. International Scientific and Practical Conference «Modern Problems of Mathematics and its Applications», Branch of Lomonosov Moscow State University in Dushanbe, Dushanbe, June 3-4, 2022;

2. Traditional International Scientific April conference dedicated to the Day of Science Workers of the Republic of Kazakhstan, Institute of Mathematics and Mathematical Modeling, Kazakhstan, Almaty, April 6-8, 2022;

3. XVII International Scientific Conference of students, undergraduates and Young scientists «Lomonosov–2022», Lomonosov Moscow State University Kazakhstan branch, Kazakhstan,Nur–Sultan, April 15-16, 2022;

4. XVII International Scientific Conference of students, undergraduates and young scientists «G'ylym ja'ne bilim-2022», L.N. Gumilev Eurasian National University, Kazakhstan, Nur–Sultan, April 11, 2022;

5.IX International Scientific Conference Problems of Differential Equations, Analysis and Algebra, Aktobe Regional University named after K. Zhubanov, Kazakhstan, Aktobe, May 24-28, 2022;

6. International Conference dedicated to the 90th anniversary of Academician Nadirov N.K., to the 80th anniversary of Academician Otelbaev M.O.

«Computational and Information Technologies in Science, Engineering and Education», Al-Farabi Kazakh National University, Kazakhstan, Astana, October 12-15, 2022;

7. International Scientific Conference dedicated to the 80th anniversary of Professor Mustafin T. G. «Actual problems of mathematics, mechanics and informatics», Karaganda University named after Academician E.A. Buketov, Kazakhstan, Karaganda, September 8-9, 2022.

8. XVIII International Scientific Conference of students, undergraduates and young scientists «Lomonosov–2023», Lomonosov Moscow State University Kazakhstan Branch, Kazakhstan, Astana, April 14-15, 2023;

9. XVIII International Scientific Conference of students, undergraduates and young scientists G'ylym ja'ne bilim-2023, L.N. Gumilyov Eurasian National University, Kazakhstan, Astana, April 12, 2023.;

10. VII World Congress of Turkic World Mathematicians (TWMS Congress-2023), K.A. Yassavi International Kazakh-Turkish University Kazakhstan, Turkestan, September 20–23, 2023.

Also, the individual results of the work were discussed in the following seminars:

1.Scientific seminar «Functional analysis and its application» (headed by academicians of the National Academy of Sciences of the Republic of Kazakhstan M. Otelbaev and R. Oinarov, professors E.D. Nursultanov, K.N. Ospanov).

2. L.N. Gumilyov Eurasian National University, Scientific seminar of the department «Basic Mathematics » (c . Astana., November 30, 2023y., April 4, April 18, 2024y.)

Compliance with the directions of scientific development or government programs

The dissertation work was carried out within the framework of a project funded from the state budget AP22684340 « On the asymptotics of attractors of the Ginzburg-Landau complex equation in a perfected domain with an oscillating boundary ».

The topic of the dissertation research corresponds to the priority direction «Intellectual potential of the country» in the field of science «Natural Sciences», the specialized scientific direction «Fundamental and applied research in mathematics, mechanics, astronomy, physics, chemistry, biology, computer science and geography».

Publications. The main results of the dissertation were published in 15 papers (5 articles and 10 papers). Of these, 4 articles in journals included in the Web of Science Core Collection and Scopus database (2 articles in journals with a percentile of more than 35), 1 article in a foreign publication, as well as 10 papers in the materials of international scientific conferences.

Description of the doctoral student's contribution to the preparation of each publication

The main results of the dissertation were published in 5 papers (1 article was written by a doctoral student alone. In 4 papers written in collaboration with scientific consultants, scientific consultants own the formulation of tasks and the choice of research methodology, and the doctoral student independently formulated the main and auxiliary results and carried out their proof).

The structure and scope of the dissertation. The dissertation consists of an introduction, four parts, a conclusion, a list of references and an appendix.

The number of sources used is 73.

Keywords: attractor, homogenization, system of Navier-Stokes equations , weak convergence, porous medium, perforated domains, rapidly oscillating terms.