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PERFECT FRAGMENTS WITH CONVEXITY CONDITION AND THEIR MODELS CLASSES

ABSTRACT

of the dissertation work for the degree of Doctor of Philosophy (PhD) in the educational program 8D05401- Mathematics

The relevance of the topic. In this dissertation, model-theoretic issues of special classes of theories are studied, which, generally speaking, are incomplete, but satisfy additional conditions. These conditions were defined by B. Jonsson and theories that satisfy these conditions were called Jonsson theories. Within the framework of the study of Jonsson theories, the concept of a fragment is naturally defined this is some theory with fixed conditions. This dissertation is devoted to the study of perfect fragments with the convexity condition and their classes of models.

The study of Jonsson theories inherently represents a class of problems related to one of the sections of mathematical discipline called model theory. This discipline is a branch of mathematical logic that studies the relationship between formal languages and their interpretations or models. It arose at the beginning of the last century at the intersection of mathematical logic, universal algebra, abstract algebra, topology and algebraic geometry. Jerome Keisler, a well-known specialist in this field, mentions two historical directions in the development of model theory: West and East. Western model theory studies complete theories, while Eastern model theory studies Jonsson theories. The conventional names “East” and “West” were formed thanks to the founders of model theory in America: A. Robinson, who lived in the eastern part, and A. Tarsky, who lived on the west coast of the USA.

When studying Jonsson theories, it was noticed that the most successful description of such theories was obtained in the case of the existence of a model companion to these theories. A Jonsson theory that has a model companion as its center is called a perfect Jonsson theory. In this case, from the class of all Jonsson theories, a subclass is distinguished that contains all perfect Jonsson theories, and describing both classes for a fixed signature is not a trivial task. Vivid examples illustrating this fact are group theory and the Abelian groups theory. The first theory (group theory) is an example of an imperfect Jonsson theory, but the Abelian groups theory, which is a subclass of the class of all groups, is already an example of a perfect Jonsson theory. In addition, if we consider the classes of models of even a perfect Jonsson theory, then we can easily construct examples when the Jonsson theory of these classes will be imperfect.

Convex theories were defined by A. Robinson, this is a subclass of inductive theories that satisfy the condition that the intersection of any two of its models is again a model of this theory, provided that this intersection is not empty. In the case where this intersection is never empty, such a theory is called strongly convex. Perfect fragments with the convexity condition are subsets of convex theories that also have the property of perfection in the Jonsson sense.

One of the syntactic invariants of an arbitrary Jonsson theory is its center: the elementary theory of a special model of a given Jonsson theory, and the model is called semantic. This model is universally homogeneous in the sense of the "Eastern" theory of models, while it is well known that the "Western" analogue of the homogeneous-universal model is a saturated model. In the case of Jonsson theory, this is not the case. If this is so, then its semantic model is saturated, which makes it possible to call perfect such Jonsson theories.

The study of perfect Jonsson theories entailed the creation of a new technical apparatus for obtaining corresponding results, generally speaking, incomplete theories, which is associated with the difficulties that arise when such theories are "poverty" and there are no analogues of known concepts and related results from the arsenal of complete theories, that is, the "Western" theory of models.

Combining the requirements of perfection and convexity for fixed fragments of a given Jonsson theory creates new opportunities for studying both "Eastern" and even "Western" theory of models. Due to the fact that any complete non-Jonsson theory can be transformed, in some enrichment of language, into a Jonsson theory using a transformation process called morlesization.

The study of properties, systematization of individual results and issues of special classes of theories, which are generally incomplete, but satisfy additional conditions, including the intersection of the concepts of theory perfection and convexity of models, were carried out by A. Tarski, A. Robinson, T.G. Mustafin, A.R. Yeshkeyev and others.

Considering the above-mentioned difficulties in studying Jonsson theories, it can be noted that this dissertation research relates to a fairly well-known and relevant topic for both "Western" and "Eastern" model theory. First of all, this work is related to the classification of Jonsson theories regarding various important model-theoretic concepts. Let us highlight the basic concepts that are relevant to this topic. An essential point is the use of the concept of convexity, both syntactic and semantic properties of various types of models. The main emphasis is on the study of countable models, namely models obtained using a certain closure operator of a fixed pre-geometry. Syntactic properties are used when specifying special types of countable models through formulas implemented by Jonsson subsets of the semantic model and various combinations of convexity, some completeness, existential simplicity and perfection of fragments of these sets. The classical concept of countable categoricity and uncountable categoricity is also explored in the framework of the study of the above fixed fragments. The countable case of categoricity was studied in the aspect of studying the concept of a holographic model. The uncountable case of categoricity was studied in connection with the concept of strong minimality of the central type of the center of a fixed Jonsson theory.

When studying Jonsson theories, as noted above, in the case of the perfectness of the considered theory, results were obtained that made it possible to transfer many relevant theorems of the “Western” model theory to the “Jonsson” formulation of these problems. At the same time, under the fixedness of this statement, in addition to the assumption of partial completeness of the theory, various combinations of requirements related to the structure of Jonsson subsets of the semantic model of these Jonsson theories were considered. During these studies, it was noticed that due to the limited completeness and non-elementary nature of the subclasses of classes of all models of these theories, the means of the theory itself are not enough to obtain the necessary results. In this regard, special enrichments of the language of the theories under consideration were considered, and in this enrichment of the initial tasks, using the concept of heredity of admissible enrichments and the new concept of the central type, a number of results were obtained, allowing us to notice progress within the framework of the study of Jonsson theories, both perfect and imperfect.

The imperfectness of Jonsson theory is clearly related to the non-elementary nature of the class of existentially closed models of this theory. Note that as a result of the morlesization process, a perfect Jonsson theory is always obtained, and the main point of this process is the enrichment of the language with a countable number of unary predicates. But the most important problem associated with the concept of heredity of permissible enrichment is the fact that at the moment there is no known description of this concept. And this open question is one of the difficult unresolved questions associated with the concept of a central type. On the other hand, knowing the properties of a counterexample for non-hereditary Jonsson theories, we can conclude that the use of the apparatus of pregeometry to study Jonsson theories is not accidental. And those additional conditions regarding the closure operator in statements related to the central type and strong minimality of the center of the Jonsson theory under consideration are determined by the model-theoretic properties of the counterexample of non-hereditary Jonsson theories.

The study of small models, and this relates specifically to countable models, is also associated with an unsolved problem within the framework of the study of Jonsson theories. It is well known that simple and atomic models are equal to each other if they are countable in the “Western” version of model theory. In the “Eastern” version, the concepts of atomicity and simplicity of the counting model are generalized and refined by the corresponding “Western” analogues. In this regard, these concepts are not equivalent, since there are corresponding counterexamples. Therefore, since at the moment, the state of these two open questions is an unsolved problem, it would be natural to consider the content of the problems of this dissertation to be quite relevant.

The goal of the work. The main goal of the dissertation research is to obtain a description of new model-theoretic concepts within the framework of studying fragments of fixed subsets of the semantic model of a given Jonsson theory.

Research objectives:

1. Obtain a theorem for the existence of a holographic model for a perfect Jonsson theory with a fixed center.

2. As part of the study of a perfect core theory, obtain a description of its core

model and the connection of this model with the Kaiser hull of such a theory.

3. As part of the study of an existentially algebraically prime theory, obtain a description of the algebraically prime models of this theory regarding the coreness of the model, provided that such a model exists.

4. As part of the study of the perfect, convex, existentially complete and existentially prime Jonsson theory, obtain a description of the core models of its center.

5. Study the model-theoretic properties of a fragment of an algebraically prime set, subject to perfectness and existential primeness with limited completeness.

6. As part of the study of fixed fragments, obtain a description of algebraically prime sets that define these fragments.

7. As part of the study of convex, perfect, existentially prime fragments with limited completeness, obtain a description of the existence of a core model of the center of this fragment.

8. Study fixed Robinson hereditary varieties with respect to algebraic closure.

9. Describe strongly minimal central types from the class of cosemanticness from a fixed Robinson hereditary spectrum of the variety.

Object of the research. fragments of fixed subsets of the semantic model of a given Jonsson theory. In particular, perfect fragments with the convexity condition.

Subject of the research: Jonsson theories, as well as fixed fragments with the convexity condition and their classes of models.

Research methodology. The dissertation work uses classical research methods in Model Theory, as well as the semantic method. The essence of this method is to transfer the first-order properties of predicate calculus from the center of the Jonsson theory under consideration to this theory itself. The semantic method is also applied to the study of Jonsson sets and their convex fragments, as well as the method of transferring first-order properties of the center of fixed fragments to the fragment itself.

Scientific novelty. In this dissertation, the model-theoretic properties of the semantic model of the theory were studied in the framework of the study of fixed fragments of the considered Jonsson theory. All concepts and the results obtained regarding these new concepts within the framework of the dissertation research are new and have not been previously considered.

Theoretical and practical value of the work. The results obtained are of a theoretical nature and can be applied in further research in the field of universal algebra and model theories, as well as in the study of closures of definable subsets of the semantic model for fixed Jonsson theories.

Since the questions that define this topic relate to classical problems of model theory, we can conclude that the scientific and applied significance is associated with all possible applications of model theory in various fields of theoretical and applied mathematics, and the results of the study can be used when teaching special courses at mathematics departments universities.

Provisions submitted for presentation. The following main provisions of the work are submitted for defense:

1. The theorem for the existence of a holographic model for a perfect Jonsson theory with a fixed center.

2. Description of the core model of a perfect core theory and the connection of this model with the Kaiser hull of such a theory.

3. Description of algebraically prime models of an existentially algebraically prime theory regarding the coreness of the model, provided that such a model exists.

4. Description of core models of the center of a perfect, convex, existentially complete and existentially simple Jonsson theory.

5. Model-theoretic properties of a fragment of an algebraically prime set, subject to perfection and existential primeness with limited completeness.

6. Description of algebraically simple sets that define fixed fragments.

7. Description of the criterion for the existence of a core model of the center of convex, perfect, existentially prime fragments with limited completeness.

8. Description of strongly minimal central types from the class of cosemantics from a fixed Robinson hereditary spectrum of the variety.

The credibility and validity of the conducted research is ensured by strict mathematical proofs and analysis of problem statements regarding well-known examples of concepts discussed in the dissertation.

Approbation of the obtained results. The main results of the dissertation were reported and discussed at the following international conferences and scientific seminars on the profile of the dissertation:

- traditional international April mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 1150th anniversary of Abu Nasir al-Farabi and the 75th anniversary of the Institute of Mathematics and Mathematical Modeling (Almaty, 2020);
- international conference “Maltsev Readings – 2020” (Novosibirsk, 2020);
- traditional international April conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan T.Sh. Kalmenova. (Almaty, 2021);
- international scientific conference dedicated to the 80th anniversary of Professor T.G. Mustafina. “Current problems of mathematics, mechanics and computer science” (Karaganda, 2022);
- international scientific and practical conference dedicated to the 105th anniversary of Doctor of Physical and Mathematical Sciences, Academician A.D. Taimanov and the 90th anniversary of West Kazakhstan University named after. M. Utemisov “Taiman Readings -2022” (Uralsk, 2022);
- VII World Congress of Mathematicians of the Turkic World “TWMS Congress-2023” (Turkestan, 2023);
- Joint meeting Seminar “Theory of Models” named after E.A. Palyutin Seminar on Model Theory (Leaders: Academician Y.L. Ershov, Doctor of Physical and Mathematical Sciences, Associate Professor S.V. Sudoplatov) Seminar “Model Theory” IMMM MES RK (Seminar on Model Theory) Head: Corresponding Member of NAS RK B.S. Baizhanov;

- scientific seminar under the guidance of Professor A.R. Eshkeev (KarU named after academician E.A. Buketov).

Publications. The main results of the dissertation were published in 12 papers. Of these, 1 article was published in a journal included in the Scopus list, 3 articles were published in journals recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan and 8 works - in materials of international scientific conferences. In work performed with co-authors, the contribution of each co-author is equal.

Structure and scope of work. The dissertation is 80 pages long and consists of the following structural elements: designations and abbreviations, introduction, three sections, conclusion, list of sources used.

In general, the dissertation consists of three sections that are interconnected. The first section presents the basic concepts and results from the Model Theory course, which are necessary for understanding the content of the dissertation. The section consists of three paragraphs, each of which contains information from classical Model theory, definitions and statements regarding the class of Jonsson theories and the special class of existentially closed theories, respectively. The concepts discussed in the first section include such concepts as model completeness, existential closure, convex theories, axiomatization of convex theories, types of atomic models, properties of amalgam and joint embedding, existentially closed models, various types of morphisms between models, various types of companions, elimination of quantifiers , pregeometry.

The second section is related to the study of the concept of holographic structure within the framework of the study of model-theoretic properties of fixed Jonsson theories in relation to the concept of holographicness of Jonsson theory and its models. The third section presents the main definitions, statements and their evidence obtained as part of the dissertation research.

Number of sources used – 70.

Keywords. Jonsson theory, existentially closed model, algebraically closed model, Robinson spectrum, Robinson hereditary variety, central type, Jonsson fragment, theoretical set, strongly minimal type, core model, holographic model.