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«MODEL-THEORETICAL PROPERTIES OF CENTRAL TYPES OF CONVEX JONSSON THEORIES»

ABSTRACT
of the dissertation submitted for the degree of Doctor of Philosophy (PhD) in the specialty «6D060100-Mathematics»

Relevance of the research. Mathematical logic is traditionally divided into four sections: model theory, set theory, recursion theory and proof theory. Jonsson theories are studied within the framework of «eastern» model theory, which in its essence was determined at one time by the problems of tasks formulated by Abraham Robinson. This name is rather arbitrary, and it only reflects the geographical place of residence for one of the founders of the model theory – A. Robinson, who lived on the east coast of the United States, while Alfred Tarski, the founder of «western» model theory, lived on the west coast of the United States. It should be noted that isomorphic embeddings and homomorphisms are studied as morphisms between models of theories in «eastern» direction, and elementary monomorphisms are studied in «western» direction. The main syntactic attribute of the «eastern» theories is their limitations regarding completeness and axiomatizability. Moreover, the length of the prenix of axioms of such theories does not exceed the number 2. The semantic feature of such theories is the closeness of models of these theories relative to chains of nested models. Typical examples of such theories are inductive theories. A subclass of the class of inductive theories that satisfies the above properties is the class of Jonsson theories. This class is illustrated by numerous classical algebraic examples that are widely used throughout mathematics. Note that in general case, Jonsson theories are not complete. Thus, solving the tasks of «eastern» model theory is significantly complicated by the fact that the modern apparatus of model theory is developed mainly for complete theories, and therefore the relevance of studying the model-theoretic properties of Jonsson theories is beyond doubt, and the development of the apparatus for researching such theories and their classes of models is obviously an interesting and complex problem of tasks of modern model theory.

Among the Jonsson theories, a special class is distinguished by means of important conditions, such as the perfectness and heredity of the Jonsson theory. Note that these conditions are determined in a completely natural way and there are a large number of algebraic examples of such hereditary perfect Jonsson theories. The perfectness of the Jonsson theory is determined by many equivalent criteria, and one of them is a fairly fruitful tool in the study of Jonsson theories. Namely, the center of such Jonsson theories is their model companion. A very important tool in studying the properties of elements of models of Jonsson theories is the concept of a central type. This concept is obtained by enriching the language of the considered theory. As it turned out, not all model-theoretic properties are preserved in the new language, in particular, for example, such a property as amalgam.

Earlier, it was also noted that such a classical concept as the stability of the theory is also not preserved in the case of an enrichment. The enrichment of the language is called permissible if it preserves the definiteness of the type of the considerated theory in the enrichment. The heredity of the Jonsson theory T tells us that with any permissible enrichment, it preserves the Jonssonness of the theory T.

This dissertation work will consist of technical and directly content parts. The technical part includes all agreements on the numbering of statements indicated in the introduction, as well as a list of designations and abbreviations given on page 3 of the dissertation. The content part consists of the introduction, three chapters that are interconnected, a conclusion and a list of sources used.

Chapter one begins with the definition of the basic concepts and necessary statements on Jonsson theories, semantic model, Jonsson and theoretical sets, fragments of the Jonsson set, Kaiser Hull of the model and Kaiser Hull of the class of models, the relationship between perfect Jonsson theory and the companion of the Jonsson theory, central types, $J-$strongly minimal formula and theory, the interdependence between strongly convex theory and J- $\varphi(x)$-convex theory.

The second chapter is a necessary and sufficient introduction to (n_1, n_2)-Jonsson theories and are introduced new concepts: (n_1, n_2)-Jonsson theory, (n_1, n_2)-positive Jonsson theory, n_1-Jonsson chain, eventually elementary chain of models. The main statements regarding the companions of these theories and chains of existentially closed models (n_1, n_2)-positive Jonsson theories are proved.

Chapter three is devoted to such concepts as essential type, essential base of central types, $APA-$set, φ(x)-convex Jonsson theory, hereditary, J-φ(x)-convex Jonsson spectrum, where the number of fragments of the perfect Jonsson spectrum of the selected subclass for the class of existentially closed models is calculated (Theorem 3.3.4), and this number of fragments is related to the estimate obtained earlier in Theorem 6 in the paper [1]. In order to take advantage of this estimate, in this chapter develops the appropriate technique for the possibility of this application.

The purpose of the work. The main purpose of the dissertation research is to obtain a description of new model-theoretical concepts and properties within the framework of the study of convex Jonsson theories, and, accordingly, fragments of fixed subsets of the semantic model of the given convex Jonsson theory.

The tasks of the research. The content of this work is the research of following tasks: the redefinition of A.D. Taimanov's questions within the framework of Jonsson theories; the description of the model-theoretic properties of some strongly convex fragments and a perfect, strongly convex Jonsson theory; the (n_1, n_2)-Jonsson theories and their model companions; the (n_1, n_2)-positive Jonsson theories and the chains of existentially closed models of these theories; the description of central types of convex Jonsson theories and the essential base of central types with strongly minimal formulas of strongly convex Jonsson theories, provided that $cl = acl = dcl$.

The object of the research. The objects of the research are Jonsson theories and the Jonsson fragments, taking into account convexity, perfectness, heredity,

and some fixed model-theoretic properties in specific cases, as well as the study of their classes of existentially closed models.

The subject of the research. The subjects of the research are Jonsson theories and their classes of models.

Research methodology. Within the framework of the conducted scientific researches, general methods of classical model theory related to the study of complete theories were applied, and methods of universal algebra are used. One of main methods of studying the Jonsson theories is the method proposed at the time by T.G. Mustafin and Yeshkeyev A.R. It consists in the fact that the elementary properties of the central replenishment are translated to the Jonsson preimage, while the central replenishment is an invariant of the Jonsson theory. The central replenishment of the Jonsson theory is the elementary theory of some model, which is called the semantic model, the existence of which is proved in the work of Morley, Vaught [2]. This model is the Jonsson analogue of the monster model, which is used in «western» model theory. In this regard, arises the problematics of studying the Jonsson companions. One of the new research methods of Jonsson theories is the method of using central types, which was proposed by Yeshkeyev A.R. The essence of this method will be to enrichment of the language of the considered Jonsson theory with constants and predicates. The resulting central type is the essence of the center of the Jonsson theory, with permissible enrichment taking into account the heredity of this theory. Thus, we move from theory to type and apply to it the technique of working with types, taken from the technical arsenal of working with types for a complete theory.

Scientific novelty. Many concepts related to the researches of convex Jonsson theories and their central types are new, due to the novelty of these concepts themselves, while their model-theoretic properties do not necessarily satisfy the corresponding properties of these concepts for complete theories. In this regard, it is possible to draw an unambiguous conclusion about the novelty of the concepts and the corresponding model-theoretic properties of these concepts.

The theoretical and practical value of the work. The work is theoretical in its content. The research of central types of convex Jonsson theories and the model-theoretic attributes associated with them can be used in further researches of the model-theoretic properties of fragments of Jonsson sets and their classes of models in classical model theory.

Since the issues that define this topic relate to classical problems of model theory, it can be concluded that scientific and applied significance is associated with all possible applications of model theory in various domains of theoretical mathematics, and the results of the research can be used when reading special courses at mathematical faculties of universities.

Provisions to be defended. The following main results of the dissertation research are submitted for defense:

1) a result has been obtained regarding the redefinition of A.D. Taimanov's questions within the framework of the Jonsson theories;

2) a result is obtained related to the model-theoretic properties of some strongly convex fragments and a perfect, strongly convex Jonsson theory;

3) the (n_1, n_2)-Jonsson theories are considered and results are obtained regarding their model companions;

4) the (n_1, n_2)-positive Jonsson theories are considered and a criterion is obtained regarding chains of existentially closed models of these theories;

5) the central types of convex Jonsson theories and an essential base of central types with strongly minimal formulas of strongly convex Jonsson theories are studied, provided that $cl = acl = dcl$. Here a result is obtained regarding the number of perfect fragments of a certain Jonsson theory and a fixed Jonsson spectrum for a fixed class of models of an arbitrary signature.

The credibility and validity of the conducted researches are ensured by the constructiveness of the methods used. General statements are formulated in the form of theorems and their proofs are presented.

Approbation of the work. The main results of the dissertation were reported and discussed at the following conferences:

– The traditional April international mathematical conference «Workshop «Problems of modeling processes in electrical contacts», dedicated to the 80th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan S.N. Kharin (Kazakhstan, Almaty, Institute of Mathematics and Mathematical Modeling, April 3-5, 2019, section «Algebra, Mathematical Logic and Geometry»);

– International scientific conference «Theoretical and applied issues of Mathematics, Mechanics and Computer Science», dedicated to the 70th anniversary of Doctor of Physics and Mathematics, Professor Ramazanov M.I. (Kazakhstan, Karaganda, KarSU named after Academician E.A. Buketov, June 12-13, 2019);

– The 16th Asian Logic Conference (Kazakhstan, Nur-Sultan, Nazarbayev University, June 17-21, 2019);

– International conference dedicated to the 10th anniversary of the Journal's release of the «Eurasian Mathematical Journal» (Kazakhstan, Nur-Sultan, Gumilyov ENU, October 16-19, 2019);

– International scientific conference «Maltsev Readings» (Novosibirsk, Russia, November 16-20, 2020);

– The traditional April international mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan T.S. Kalmenov (Kazakhstan, Almaty, Institute of Mathematics and Mathematical Modeling, April 5-8, 2021, section «Algebra, Mathematical Logic and Geometry»);

– International scientific conference «Maltsev Readings» (Novosibirsk, Russia, September 20-24, 2021);

– International scientific conference «Actual Tasks of Mathematics, Mechanics and Computer Science» dedicated to the 80th anniversary of Professor T.G. Mustafin (Kazakhstan, Karaganda, September 8-9, 2022);

– International scientific conference «Mathematical Logic and Computer

Science» (Astana, Kazakhstan, October 7-8, 2022);

– III International «Taimanov readings» dedicated to the 85th anniversary of the University, «Modern Mathematics: problems and applications» (Kazakhstan, Kyzylorda, November 25, 2022).

The results were also reported and discussed at the scientific seminar of the Department of Algebra, Mathematical Logic and Geometry named after Professor T.G. Mustafin of the E.A. Buketov KarSU (head – Doctor of Physics and Mathematics, Professor A.R. Yeshkeyev).

The publication of the results. The main results of the dissertation were published in 18 papers, of which 2 articles were published in journals included in the Scopus databases (in the Lobachevsky Journal of Mathematics, Percentile – 55, in the Bulletin of the Karaganda University-Mathematics, Percentile – 35); 5 articles were published in journals recommended by the Committee for Quality Assurance in the sphere of the Education and Science for the Ministry of Science and Higher Education of the Republic of Kazakhstan; 11 scientific work were published in the materials of international scientific conferences.

In the works performed with co-authors, the contribution of each of the co-authors is equal.

The structure and scope of the dissertation. The volume of the dissertation is 68 pages. The work consists of the following structural elements: designations and abbreviations, an introduction, the main three sections, a conclusion and a list of sources used. The numbering of definitions and statements has three indexes: the first index is the number of the section, the second is the number of the subsection, and the third is the own number of the definition or statement in this subsection.

The number of sources used is 70.

Keywords. Jonsson theory, perfect hereditary Jonsson theory, $\varphi(x)$-convex theory, J-φ(x)-convex theory, Jonsson fragment, core model, central type, strongly minimal formula and strongly minimal theory, APA −set, essential base, perfect hereditary Jonsson spectrum.