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MODEL-THEORETICAL PROPERTIES OF COMPANIONS OF HYBRIDS OF JONSSON THEORIES

ANNOTATION

of the dissertation work for the degree of Doctor of Philosophy (PhD)
specialty 6D060100-Mathematics

The relevance of the topic. This work represents one of the latest main trends in Model Theory. Model Theory today is a separate branch of mathematical science. Like any science, Model Theory has its own field of study. The main objects of research are models (structures, algebraic systems). The main research tool is considered to be the language of mathematical logic, namely predicate calculus. The main concept borrowed from mathematical logic is formal language.

A well-known scientist in this field, J. Keisler, divided the Model Theory into two essential directions: "Eastern" and "Western". These names are of a conditional nature. That is, the first creators of Model Theory, A. Robinson and A. Tarski, carried out their professional careers on the eastern and western coasts of the United States, respectively. Each of the mentioned directions has its own characteristics: the "Western" direction often began with problems of mathematical analysis and set theory, the "Eastern" direction often began with problems of universal algebra.

The result of this dissertation work, due to its content, refers to the Model Theory of the "eastern" direction. The objects of this direction are Jonsson theories and classes of their models. The most unusual of these objects is: theories, generally speaking, are incomplete, and some subclasses of the models under consideration are not even elementary, but since studying such theories is generally very difficult, specialists always consider the scope of research in these two cases with the following conditions. Complementary theories as well as classes of their existentially closed models, become elementary when these theories are perfect.

One of the new directions in the study of Jonsson theories is the topic of hybrids of Jonsson theories. This dissertation work considers the model-theoretic properties of a new class of Jonsson theories, namely theories obtained using various algebraic constructions of semantic models of two completely different Jonsson theories of the same language. Such theories were called hybrids.

The dissertation work considers the model-theoretical properties of various companions of fixed hybrids. Such properties of theories include almost all the classical attributes of modern Model Theory, such as stability, categoricity, rigid minimality, model completeness, axiomatization, interpretation, spectral problems, etc. Regarding the semantic aspect, we are interested in various properties associated with the concept of definable formulaic subsets of the semantic model of hybrid in relation to the following concepts: atomicity, algebraic primeness, existential closureness, convexity. Another unique feature of theories is existential primeness.

According to this work, these conditions are always met for the objects under consideration.

Considering the above, the relevance and novelty of this topic is beyond doubt.

The goal of the work. The main goal of the dissertation work is to study the model-theoretic properties of the companions of the hybrids of Jonsson theories. At the same time, syntactic and semantic similarities of hybrids, hybrids of positive Jonsson theories, special subclasses of small models of hybrids of Jonsson theories are studied.

Research objectives:

Hybrids of Jonsson theories, since they represent a new concept, still remain largely unexplored objects. Our main goal is to study this new object as much as possible based on its model-theoretic properties. In this direction, we tried to study hybrids of Jonsson theories within the framework of the following well-known model-theoretic concepts. If we list these properties, they are mentioned in the previous paragraph. However, in this dissertation work we examined the following results:

1. To consider model consistentcy and ω-categoricity in the class of existentially prime convex $\forall\exists$-complete Jonsson theories;

2. To obtain a result related to the model-theoretic properties of special subsets of the semantic model of some fixed Jonsson theory;

3. To consider the model consistentcy and ω-categoricity of hybrids of fragments of positive Jonsson theories.

4. To consider the feature of the connection between the outer and inner worlds for two existentially closed models of Jonsson theory;

5. To obtain a criterion for uncountable categoricity in the language of strong minimality of the central type of the hereditary cosemantic class of a fixed Robinson spectrum;

6. To obtain a criterion for the syntactic similarity of hybrids of cosemantic classes of the Jonsson spectrum of an axiomatizable class of models of a countable language of a fixed signature;

7. To study the condition for the existence of some syntactically similar theory of polygons for any perfect Jonsson hybrid.

Object of the research is hybrids of Jonsson theories and their classes of models. In particular, the model-theoretic properties of the companions of hybrids of Jonsson theories.

Subject of the research: Jonsson theories and their related classes of models.

Research methodology.

The dissertation research methods include all the classical methods of Model Theory, as well as the semantic method, which has recently been rapidly developing. The meaning of this method is to copy the properties of first-order predicate calculus into this very theory of the Jonsson theory under consideration. In addition, semantic methods are used to study the companions of the hybrids of Jonsson theories.

Scientific novelty.

Since all concepts related to this topic are newly defined concepts, all obtained and published results have no analogues due to the generality of the objectives of this dissertation.

Theoretical and practical value of the work.

First of all, the results obtained are of theoretical significance in their content. Secondly, the results obtained in the dissertation may well contribute to the development of the foundations of Model Theory. After all, all the tasks in this dissertation, having an "eastern" direction, largely mean generalization and clarification.

Provisions submitted for presentation.

The following are submitted for presentation:

1. Model consistentcy and ω-categoricity in the class of existentially prime convex $\forall\exists$-complete Jonsson theories are considered.

2. A result is obtained related to the model-theoretic properties of special subsets of the semantic model of a some fixed Jonsson theory. The hybrid of the fragments under consideration has a model that contains a special core subset, a definable closure, which is given by some existentially closed model, which is an algebraic prime model of the theory under consideration.

3. The model consistentcy and ω-categoricity of hybrids of fragments of positive Jonsson theories are considered.

4. The feature of the connection between the outer and inner worlds is considered for two existentially closed models of Jonsson theory.

5. A criterion for uncountable categoricity is obtained in the language of strong minimality of the central type of the hereditary cosemantic class of a fixed Robinson spectrum.

6. A criterion for the syntactic similarity of hybrids of cosemantic classes of the Jonsson spectrum of an axiomatizable class of models of a countable language of a fixed signature is obtained.

7. The condition for the existence of some syntactically similar theory of polygons for any perfect Jonsson hybrid is studied.

The credibility and validity of the conducted researches are ensured by the constructiveness of the methods used. General statements are formulated in the form of theorems and their proofs are presented.

Approbation of the work.

The main results of the dissertation were reported and discussed at the following conferences and seminars:

- - VII Franco-Kazakh Colloquium in Model Theory (14-18 November 2022, Camille Jordan Institute, Claude Bernard Lyon University, Lyon, France);

- Logic Colloquium 2019: European Summer Meeting of the Association for Symbolic Logic (ASL) (11-16 August 2019, Prague, Czech Republic);

- Logic Colloquium 2021: European Summer Meeting of the Association for Symbolic Logic (ASL)(19-24 July 2021, Poznan, Poland);

- Logic Colloquium 2022: European Summer Meeting of the Association for Symbolic Logic (ASL) (June 27 – July 1, 2022, Reykjavik, Iceland);

- 16th Asian Logic Conference (17-21 June 2019, Nur-Sultan, Republic of

Kazakhstan);

- Maltsev readings: international conference (November 19-22, 2018, Novosibirsk, Russia);
- Maltsev readings: international conference (November 16-20, 2020, Novosibirsk, Russia);
- Maltsev readings: international conference (November 20-24, 2021, Novosibirsk, Russia);
- Traditional international April mathematical conference (April 3-5, 2019, IMMM MNVO RK, Almaty, Republic of Kazakhstan);
- Traditional international April mathematical conference (April 2020, IMMM MNVO RK, Almaty, Republic of Kazakhstan);
- Traditional international April mathematical conference (April 2021, IMMM MNVO RK, Almaty, Republic of Kazakhstan);
- International conference dedicated to the 10th anniversary of the publication of the journal "Eurasian Mathematical Journal" (October 16-19, 2019, Nur-Sultan, Republic of Kazakhstan);
- International scientific conference "Mathematical logic and computer science" (October 7-8, 2022, Astana, Republic of Kazakhstan);
- International scientific and practical conference "Taiman Readings – 2022", dedicated to the 105th anniversary of Doctor of Physical and Mathematical Sciences, Academician A.D. Taimanov and the 90th anniversary of West Kazakhstan University named after. M. Utemisov (November 30, 2022, Uralsk, Republic of Kazakhstan);
- IX International Conference (May 24-28, 2022, Aktobe Regional University named after K. Zhubanov, Aktobe, Republic of Kazakhstan);
- International Scientific Conference "Actual Problems of Mathematics, Mechanics and Informatics", dedicated to the 80th anniversary of Professor T.G. Mustafin (September 8-9, 2022, Karaganda University named after Academician E.A. Buketov, Karaganda, Republic of Kazakhstan);
- XV International Scientific Conference of Students and Young Scientists "ǴYLYM JÁNE BILIM - 2020", (2020, Nur-Sultan, Republic of Kazakhstan);
- The reports were made in the "Mathematical Logic" laboratory at a seminar at the Institute of Applied Mathematics of Karaganda University named after academician E.A. Boketov.

Publications. The main results of the dissertation were published in 28 works: 1 article in journal included in the Scopus list, 6 articles were published in journal recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan and 21 works in the materials of international scientific conferences.

In the works performed with co-authors, the contribution of each of the co-authors is equal.

The structure and scope of the dissertation.

A dissertation of 83 pages consists of the following structural elements: introduction, two sections, conclusion, list of sources used.

The dissertation consists of two sections, which are interconnected. The first section of the dissertation presents definitions and theorems of the basic concepts of Model Theory, necessary to describe the model-theoretic properties of Jonsson theories. In this section, in the framework of the study of Jonsson theories, which, generally speaking, are incomplete, classical issues of Model Theory, such as categoricity, completeness, syntactic and semantic similarity, are considered. In particular, the questions of countable and uncountable categoricity of the cosemantic class of the Jonsson spectrum of an arbitrary model of the signature under consideration are studied. We also consider a whole class of theories associated with the class of fixed models of incomplete theory, which is called the Jonsson spectrum of this class. These theories differ from each other up to some binary relation, which is a generalization, and in some cases even a refinement of the classical concept of an equivalence relation between models. In addition, the work presents theorems related to the classical theorems of Model Theory (M. Morley, D. Saracino and P. Lindström) in the framework of the study of Jonsson spectra of special classes of structures. Since a new and relevant method for studying Jonsson theories is to study theories using syntactic and semantic similarities, this section considers syntactic and semantic similarities for complete theories and for Jonsson theories.

The second section of the dissertation is the main one. This section considers the model-theoretic properties of hybrids of Jonsson theories, i.e., small models of hybrids of special subclasses of Jonsson theories, hybrids of positive Jonsson theories, the geometry of theoretical sets of fragments of hybrids and syntactic and semantic similarities of hybrids are described. At the same time, model-theoretic properties of the algebra of central types of mutually model-compatible fragments are considered. This section describes the outer and inner worlds of the existentially closed model of Jonsson theory, the λ-comparison of the two existentially closed models, and the algebra of the central type.

The number of sources used is 71.

Keywords. Jonsson theory, semantic model, existentially closed model, existentially prime theory, model companion, hybrid of Jonsson theories, fragment, model-consistent theory.