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«JONSSON PAIRS IN PERMISSIBLE ENRICHMENTS»

ABSTRACT
of the dissertation work for the degree of Doctor of Philosophy (PhD) in the educational program 8D05401- Mathematics

The relevance of the topic. This dissertation concerns Model theory, which belongs to the department of mathematical logic. Model theory emerged as a separate science in the 50s of the last century. Scientists like Keisler and Chen, in their famous book "Model Theory", said that model theory is the bridge between universal algebra and mathematical logic. Historically, the development of model theory has been closely connected with the names of mathematicians and logicians from different countries for several generations. We all know that the two main founders of model theory in the United States, Alfred Tarski and Abraham Robinson, can be divided into "Eastern" and "Western" models depending on their geographical location. These names, of course, are arbitrary, and the topic of my dissertation is related to the "Eastern" direction of model theory. A. Robinson, the founder of the theory of the "origin" of models, identified and highlighted the foundations and tasks of this direction.

It should be noted that Jonsson theories inherently represent a specific subclass of the class of inductive theories, generally speaking, incomplete. That is, when studying Jonsson theories in relation to complete theories studied by the theory of "Western" models, we cannot use concepts and results obtained from the arsenal of complete theories since they are obviously incomplete. In this regard, the problem of redefining and searching for analogues of concepts and results in complete theories, respectively, is relevant and exciting but, at the same time, quite a difficult task.

The relevance of my dissertation work is also related to studying model-theoretic properties of the universal Jonsson algebra (i.e., satisfying the Jonsson conditions). Since Jonsson theory is an incomplete theory, and research methods are not as developed as for the complete theory, therefore, any result obtained for the incomplete theory is new. From the point of view of model theory, the direction and results of the later Jonsson theories are considered more general than the model of the classical theory. The search for solutions to questions aimed at enriching the language of the established Jonsson theory is associated with the problems of Jonsson theories. It is known that there are examples showing that the signature of a given Jonsson theory is not preserved when the signature of a given Jonsson theory is enriched with one predicate.

A still unsolved problem is the problem of characterizing the concept of heredity in Jonsson theory. The following important counterexample confirms the relevance of this problem: the elementary theory of an algebraically closed field, after being enriched with a unary predicate, ceases to be jonssonness. In this regard,

studying model-theoretic properties of central types in predicate enrichment represents an important model-theoretic task for describing the hereditary Jonsson theories.

In this dissertation, the study of the model-theoretic properties of the concept of heredity using the central type is considered not only on theories but also on the Jonsson spectrum. The concept of the Jonsson spectrum contains syntactic and semantic invariants of all classes of its existentially closed models. Moving on to the study of cosemantic classes of this spectrum, we begin to study those Jonsson theories with the same semantic model. With a permissible enrichment of the language of a given spectrum, the problem of heredity remains the same for the spectrum. Therefore, having solved the problem for the theory, we can consider the more complex question of heredity of the Jonsson spectrum.

At the same time, another pressing issue in the work, which can be called the beginning of our research, was the determination of the properties of Jonsson semantic pairs. So far, it is known that elementary pairs have been proven to be complete. Now, having considered these pairs in an admissible enrichment, we will receive an answer to the question: does the theory of such a pair preserve completeness, or is it jonssonness?

The goal of the work. The main goal of the dissertation research is to describe the syntactic and semantic properties of given hereditary Jonsson theories, pairs and central types of Jonsson spectrum with admissibility in enrichment. We will deal with a valid enrichment of a language signature that preserves type definability with an enriched version of stability. The main syntactic object of study will be the concept of heredity of Jonsson theory. Recall that we will call a Jonsson theory hereditary if it remains jonssonness for a fixed permissible enrichment. We should also note that such syntactic and semantic properties of theories, spectrum and their classes of models concerning such concepts as convexity, completeness, model completeness, existential simplicity, categoricity, model compatibility, model companion, finite cover property, strongly minimal will be studied in the language of central types in the corresponding enrichments.

Research objectives. The content of this work is to study the following tasks: to obtain some model-theoretic properties of fragments of special subsets within the framework of studying the perfect hereditary Jonsson theory in permissible enrichments; to obtain a criterion for ω-categoricity in the study of modular convex and $\forall\exists$-complete Jonsson theories; to obtain some model-theoretic properties of the heredity of theories within the framework of the study of hereditary Jonsson theories consisting of one signature in permissible enrichments; to obtain the properties of orbital types and strongly minimal sets that define a special Jonsson geometry and are specified by the closure operator; to obtain results associated with J-strongly minimal types in the semantic model; to obtain some model-theoretic properties of J-λ-stable and hereditary theories; to prove a generalization of stability for the Jonsson hereditary spectrum, connecting J-λ -stability and classical stability; to obtain the basic properties of semantic pairs and e.f.c.p. for a J-λ -stable perfect Jonsson spectrum.

Object of the research. The object of study is Jonsson theories, as well as J -stable and hereditary Jonsson theories and their classes of models. In addition, the syntactic object of our research is the heredity of Jonsson theories.

Subject of the research. The subject of the study is Jonsson theories and their classes of models.

Research methodology. The Karaganda school of model theory, founded by Professor A.R. Yeshkeyev, has developed new methods for studying Jonsson theories, a unique feature of which is the use of working techniques of central types in permissible enrichment. The essence of using the method of central types is to enrich the language of the theory under consideration with constants and predicates. And in the case of incomplete theories, the classical approach of model theory does not work, therefore, when studying the dissertation work, a semantic approach was used.

Scientific novelty. In the dissertation we consider Jonsson theories related to stability. By now it has been proven that the elementary pairs are complete, and further we consider Jonsson pairs in a permissible enrichment for the incomplete theory.

Theoretical and practical value of the work. The scientific and applied significance of this dissertation is of interest in such theoretical sciences as the theory of models and universal algebra, as well as in applications of various related classes of mathematics and computer science. For example, formalizing ontologies of different subject areas of artificial intelligence is very important when working with databases.

Provisions submitted for presentation. The following main results of the dissertation research are submitted for defense:

1) Some model-theoretic properties of fragments of special subsets within the framework of studying the perfect hereditary Jonsson theory in permissible enrichments;
2) A criterion for ω-categoricity is obtained within the framework of the study of modular convex and $\forall\exists$-complete Jonsson theories;
3) Some model-theoretic properties of hereditary theories were obtained by studying hereditary Jonsson theories consisting of one signature in permissible enrichments;
4) The properties of orbital types and strongly minimal sets that define a special Jonsson geometry and are specified by the closure operator are obtained;
5) A criterion for cosemantic of semantic pairs for the Jonsson spectrum is obtained;
6) Results related to J-strongly minimal types in the semantic model were obtained;
7) Some model-theoretic properties of J-λ-stable and hereditary theories are obtained;
8) A generalization of stability for Jonsson hereditary spectrum has been proven, connecting J-λ -stability and classical stability;
9) The basic properties of semantic pairs and e.f.c.p. are obtained for a J-λ -stable perfect Jonsson spectrum.

The credibility and validity of the conducted research is ensured by strict mathematical proofs and analysis of problem statements regarding well-known examples of concepts discussed in the dissertation.

Approbation of the obtained results. The main results of the dissertation were presented and discussed at the following international conferences and seminar:

- Logic Colloquium 2023: European Summer Meeting of the Association for Stmbolic Logic (ASL) (5–9 June 2023, Milan University, Milan, Italy);
- 16th Asian Logic Conference (17-21 June 2019, Nur-Sultan, Republic of Kazakhstan);
- Maltsev readings: international conference (November 19-22, 2018, Novosibirsk, Russia);
- Traditional international April conference (April 3-5, 2019, Institute of Mathematics and Mathematical Modeling, Almaty, Republic of Kazakhstan);
- Traditional international April conference (April 6-8, 2022, Institute of Mathematics and Mathematical Modeling, Republic of Kazakhstan);
- Traditional international April conference (April 16-19 and 22, 2024, Institute of Mathematics and Mathematical Modeling, Republic of Kazakhstan);
- International conference dedicated to the 10th anniversary of the publication of the journal "Eurasian Mathematical Journal" (October 16-19, 2019, Nur-Sultan, Republic Kazakhstan);
- International scientific conference "Theoretical applied problems of mathematics, mechanics and computer science" (June 12-13, 2019, Karaganda University named after academician E.A. Buketov, Karaganda, Republic of Kazakhstan).
- The reports were made in the "Mathematical Logic" laboratory at a seminar at the Institute of Applied Mathematics of Karaganda University named after academician E.A. Buketov.

Publications. The main results of the dissertation were published in 12 works: 1 article - in a journal included in the Scopus database; 3 articles - in journals recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan; 8 works – in materials of international scientific conferences.

In work performed with co-authors, the contribution of each co-author is equal.

Structure and scope of work. A dissertation of 64 pages consists of the following structural elements: introduction, two chapters, conclusion, bibliography. The numbering of definitions and conclusions consists of three indexes: the first index is the section number, the second is the paragraph number, the third is the unique number of the definition or theorem in this paragraph.

The dissertation consists of two closely related sections. The first section describes the main results that capture some properties of hereditary theories in permissible enrichment, and the basic concepts used in obtaining these results. To discuss these results, the first section is divided into six paragraphs, and here concepts such as perfect Jonsson theories and spectrum, Jonsson stability and some of its generalizations, notion of permissible enrichment and central types, strongly minimal sets in the prime geometry and essential types of the semantic model fixed

Jonsson theory, categoricity properties of Jonsson theory, axiomatic representation of forking in Jonsson theories.

The second section presents the main results of the dissertation research, in particular, the properties of permissible Jonsson pairs are considered. This section consists of four paragraphs. In each section, concepts are considered and statements are proven regarding properties of P-stability for hereditary Jonsson theories, semantic pairs, existentially finite cover property, and J-non-multidimensionality for hereditary Jonsson theories.

Number of sources used is 64**.**

Keywords. Jonsson theory, semantic model, perfect hereditary Jonsson theory, perfect hereditary Jonsson spectrum, modular pregeometry, permissible enrichment, central type, categoricity, existentially closed model, J-strongly minimal set, essential type, J-stable theory, semantic pairs, existentially finite cover property, J-non-multidimensionality.