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WEIGHTED ESTIMATES OF QUASILINEAR AND BILINEAR
DISCRETE OPERATORS

ABSTRACT

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in the educational program 8D05401 –Mathematics

Relevance of the topic. The dissertation is devoted to the problems of obtaining weighted estimates of discrete Hardy-type operators.

The famous Hardy inequalities, given in continuous and discrete forms, have enormous applications in mathematical analysis, the theory of differential equations, and other areas of mathematics. The initial type of these inequalities was later significantly generalized. The Hardy inequality has become a separate major direction in mathematics.

One of the main problems in the theory of weighted inequalities is obtaining criteria for fulfilling continuous and discrete inequalities involving Hardy-type operators. It is obvious that the weighted estimates of quasilinear, bilinear integral and discrete operators involving Hardy operators are characterization Hardy-type inequalities. In addition, considering such discrete inequalities for a class of Matrix operators can be called a new wave of research. In general, since the boundedness criterion of a discrete operator with a matrix in the weighted Lebesgue space is not defined, different conditions are imposed on the matrix, and wider results are obtained than in the case without a matrix.

Hardy type integral inequalities are often studied first in mathematics. However, the technique and methods for proving integral and discrete inequalities are different. In addition, the discrete inequality has a wider range of parameters to study than the integral inequality, since Hardy-type integral inequalities hold only in trivial cases, that is, in the case $0 < p \leq 1$ the integral inequality only holds when the left-hand side is zero. And the discrete analog is executed in case $0 < p \leq 1$. In this dissertation, we present results for case $0 < p \leq 1$, which are new.

In the thesis, the Hardy operator and three-parameter discrete bilinear and quasilinear weighted inequalities involving the matrix operator were studied.

Integral and discrete Hardy-type inequalities and their various generalized forms are studied in many countries, such as Russia, Sweden, Czech Republic, USA, Croatia, Italy, Norway, and China.

The study of inequalities of Hardy-type in Kazakhstan began with the works of M. Otelbaev and R. Oinarov and was continued in the works of K. Myrzataeva, A. Kalybay, A.M. Temirkhanova, A. Abylaeva. On the topic of this dissertation work, earlier there were dissertations of S.Kh. Shalginbayeva, A.M. Temirkhanova Zh.A. Taspaganbetova and B. Omarbaeva.

The purpose of the study: To obtain the necessary and sufficient conditions for the fulfillment of the discrete weighted Hardy-type inequality for quasilinear, quasilinear and bilinear operators with a matrix in different relations of parameters.

Research objectives:

- obtaining the necessary and sufficient conditions for the fulfillment of discrete quasilinear weighted inequalities in some relationship of the parameters in terms of weighted sequences, and obtaining a two-sided estimate of the smallest constant of the inequality;

- on the basis of the condition imposed on the matrix, obtaining the necessary and sufficient conditions for the fulfillment of discrete quasilinear weighted inequalities involving the matrix operator in the possible relationship of the space parameters in terms of weighted sequences and matrix elements, and obtaining a two-sided estimate of the smallest constant of the inequality;

- on the basis of the condition imposed on the matrix, obtaining the necessary and sufficient conditions for the fulfillment of discrete bilinear weighted inequalities involving the matrix operator in the possible relationship of the space parameters in terms of weighted sequences and matrix elements, and obtaining a two-sided estimate of the smallest constant of the inequality.

The objects of research. Iterated and bilinear discrete inequalities involving Hardy-type operators in the Lebesgue space of sequence.

Research methods. Results related to well-known two-weighted Hardy-type inequalities and the method of dividing sequences into parts, the "localization method", are used to obtain the main results. In addition, the "iteration method" is also used during the study of bilinear inequality.

Scientific novelty. Criteria for fulfillment of weighted Hardy-type inequalities for some class of discrete quasilinear and bilinear operators.

Results presented to the defense. The following results were obtained in the dissertation work:

- in the case of the parameter $0 < q < \min\{p, r\} < \infty$, necessary and sufficient conditions for the fulfillment of Hardy-type inequalities for discrete quasilinear operators were obtained;

- in the case of the parameter $0 < r < p \leq q < \infty$, $0 < p \leq 1$, necessary and sufficient conditions for the fulfillment of Hardy-type inequality for a discrete quasilinear operator were obtained;

- in the case $0 < p \leq q < \infty$, necessary and sufficient conditions for the fulfillment of Hardy-type inequalities involving discrete quasilinear operators with the matrix were obtained;

- in the case of the parameter $0 < r < p \leq q < \infty$, $p > 1$, necessary and sufficient conditions for the fulfillment of Hardy-type inequality involving discrete quasilinear operator with the matrix were obtained;

- in the case of the parameter $0 < \min\{p, s\} < \max\{p, s\} \leq q < \infty$, criteria for the fulfillment of Hardy-type inequality for a discrete bilinear operator with the matrix were obtained;

- in the case of the parameter $0 < \min\{p, s\} \leq 1 < \max\{p, s\} \leq q < \infty$, criteria for the fulfillment of Hardy-type inequality for a discrete bilinear operator with the matrix were obtained;

- in the case of the parameter $1 < \min\{p, s\} \leq q < \max\{p, s\} < \infty$, criteria for the fulfillment of Hardy-type inequality for a discrete bilinear operator with the matrix were obtained.

The theoretical and practical value of the results. The results of the dissertation contribute greatly to the development of the theory of functional analysis, including the theory of iterated and bilinear discrete Hardy-type inequalities.

Approbation of the dissertation results. The main results of the dissertation were presented at the following conferences:

1. Traditional international scientific conferences of the Institute of Mathematics and Mathematical Modeling (Almaty, 2022, 2023).

2. XVII International Scientific Conference for students and young scholars «GYLYM JÁNE BILIM - 2022» (Astana, 2022).

3. XVII International Scientific Conference of Students, Masters and Young Scientists “LOMONOSOV – 2022” (Astana, 2022).

4. International Conference «Computational and Information Technologies in Science, Engineering and Education» (CITech-2022) dedicated to the 90th anniversary of Academician N. K. Nadirov, to the 80th anniversary of Academician M. O. Otelbaev (Almaty, 2022).

5. International Mathematical Conference “Functional Analysis in Interdisciplinary Applications” (Antalya, 2023).

Individual results of the dissertation work:

- three times performed at the scientific seminar "Functional analysis and its application" (supervisors of the seminar were academicians of NAS RK M.Otelbaev and R. Oinarov, professors E.D. Nursultanov, K.N. Ospanov);

- repeatedly presented and discussed at the scientific seminar "Weighted inequalities and their applications" (supervisors of the seminar were academician of NAS RK R. Oinarov, associate professors A.M. Temirkhanova, A.M.Abylayeva, M. Alday).

Publications. Based on the results of the dissertation work, 13 papers were published, namely 5 papers in ranked journals indexed in Scopus or Web of Science databases (including 2 articles in scientific journals included in the list recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan) and 8 papers in international scientific conference proceedings.

The structure and scope of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion and a list of references. The total volume of the dissertation is 106 pages.

In the first chapter, we present well-known statements and lemmas necessary to prove the main results, and also obtain necessary and sufficient conditions for

the fulfillment of weighted estimates of quasilinear discrete operators in the following relationships of the parameter:

- $0 < q < \min\{p, r\} < \infty, p > 1$;
- $0 < q < \min\{p, r\} < \infty, 0 < p \leq 1$;
- $0 < r < p \leq q < \infty, 0 < p \leq 1$.

In the second chapter, necessary and sufficient conditions for fulfilling weighted estimates of quasilinear discrete operators with a matrix satisfying discrete Oinarov condition are obtained for the following parameter relations:

- $0 < p \leq q < \infty, p > 1, 0 < r < \infty$;
- $0 < p \leq q < \infty, 0 < p \leq 1, 0 < r < \infty$;
- $0 < p \leq q < \infty, 0 < p \leq 1, 1 < r < \infty$;
- $0 < p \leq q < \infty, p > 1, 1 < r < \infty$;
- $0 < r < p \leq q < \infty, p > 1$.

In the third chapter, criteria for fulfilling weighted estimates of a bilinear discrete operator with a matrix satisfying discrete Oinarov condition are obtained for the following parameter relations:

- $0 < p, s \leq 1, 0 < \min\{p, s\} < \max\{p, s\} \leq q < \infty$;
- $1 < p, s < \infty, 0 < \min\{p, s\} < \max\{p, s\} \leq q < \infty$;
- $0 < \min\{p, s\} \leq 1 < \max\{p, s\} \leq q < \infty$;
- $1 < \min\{p, s\} \leq q < \max\{p, s\} < \infty$.

In conclusion, the main conclusions are formulated and the scope of their application is described.

The number of sources used is 74.

Keywords. Weighted Lebesgue space, discrete operator, matrix operator, quasilinear operator, bilinear operator, weighted estimate, Hardy-type inequality.