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MODEL THEORETIC PROPERTIES OF COUNTABLE MODELS OF
EXISTENTIALLY PRIME JONSSON THEORIES

ABSTRACT

of the dissertation work for the degree of Doctor of Philosophy (PhD)
specialty «6D060100-Mathematics»

The relevance of the topic. The study of the most important syntactic and semantic properties of special countable models that satisfy the condition of atomicity or primeness in the class of existentially closed models of a fixed inductive theory is one of the classical directions in the general model theory associated with the description of small models. Small models include models such as algebraically prime, atomic, core, rigid and minimal. One of the unresolved issues in this part of Model Theory is R. Vaught's conjecture for countable models.

On the other hand, in cases where there are atomic or countable models of countable power for an incomplete theory, the question arises of describing the total number of models of this theory. Thus, for incomplete theories, the significance of these issues related to the description of small models contains a large number of open questions related to their description. In this regard, the relevance of these tasks is beyond doubt.

The goal of the work. The main goal of the dissertation is to study the most important syntactic and semantic properties of special counting models that satisfy the condition of atomicity or primeness in the class of existentially closed models of a fixed inductive theory. Moreover, these models are obtained as a result of the closure of the corresponding definable subsets of a certain semantic model. In particular, conditions have been found under which such closures make it possible to find among the corresponding small models for which the concepts of atomicity and primeness coincide as in the case of countable models of complete theories.

The tasks of the research. The classical result of R. Vaught is well known related to the description of countable models, which, both syntactically and semantically, are in a sense the "smallest" models of the theory under consideration.

This result, a criterion, connects the concepts of the atomic and prime models when they are countable. But at the same time, the theory under consideration is complete. In this dissertation, all results related to obtaining descriptions and characteristics of countable models belong to the class of generally speaking incomplete theories, namely fixed Jonsson theories. In the famous work of J. Baldwin and D. Kueker "Algebraic prime models," analogues of the concepts of countable-atomic and countable-prime models were defined within the framework of the study of the class of generally speaking incomplete inductive theories.

In the same work, we tried to formulate many open questions related to the above criterion of R. Vaught in the general case, the authors J. Baldwin and D. Kueker failed to obtain a result connecting the new concepts of atomicity and primeness of models in a general form. Thus, in the case of Jonsson theories, which

are a special case of inductive theories, the above problems of describing atomic and prime models within the framework of the work of J. Baldwin and D. Kueker is an urgent and unsolved problem.

In the dissertation, the main task is to clarify the concepts of atomicity and primeness of countable models of inductive theories from the work of J. Baldwin and D. Kueker within the framework of the study of Jonsson theories.

For this purpose, the concepts of atomicity and primeness of countable models were defined using the closure operator by a given pregeometry on the set of all subsets of the semantic model of the considered Jonsson theory and the use of the well-known results of the work of J. Baldwin and D. Kueker within the framework of refined concepts of atomicity and primeness of countable models from the work "Algebraic prime models" by J. Baldwin and D. Kueker.

In addition, countable models were considered, which are varieties of core and rigid models in the class of existentially closed models of the considered Jonsson theory, and they were also obtained as a result of the closure of some fixed definable subsets of the semantic model of the considered Jonsson theory. Thus, we can notice that the problems considered in the dissertation are a clarification and, in some cases, a generalization of the issues considered in the work "Algebraically prime models" by J. Baldwin and D. Kueker and related to new formulations of problems connecting three topics: description of countable models of incomplete theories, a description of these theories regarding the existence of "small" models and a study of the properties of the closure operator with respect to the above models.

The object of the research. The theoretical set of a fixed Jonsson theory will be considered as a syntactic object of such models. In some cases we are dealing with the closure of definable subsets that generate the desired model, and a fragment of such subsets is a Jonsson theory. Considering special formulaic subsets, which, on the one hand, define atomicity in the sense of the results of J. Baldwin and D. Kueker in the work "Algebraic prime models", but on the other hand, firstly, give some geometric interpretation in the sense of pregeometry of the semantic model specified on the Boolean, secondly, they provide a new tool for studying the corresponding type of atomicity.

The subject of the research. Study of the most important syntactic and semantic properties of special countable models that satisfy the condition of atomicity or primeness in the class of existentially closed models of a fixed inductive theory.

Research methodology. The dissertation research methods include many classical methods of Model Theory, as well as a rapidly developing model-theoretic approach to the study of formula-definable subsets of some fairly large model under consideration. Research methods include not only classical methods known previously, but also completely new methods that emerged in the study of parallel problems from the general topics of Model Theory and Universal Algebra.

Scientific novelty. This work provided the basis for generalizing the concepts of countably prime and countably atomic models within the framework of the following modifications. Firstly, the concept of an elementary embedding was replaced by an isomorphic embedding, and thus the authors of the work "Algebraically prime models" J. Baldwin and D. Kueker, instead of a prime

model, began to work with the concept of an algebraically prime model. Secondly, the concept of countable atomicity has changed from the concept of countable atomicity for complete theories. One of the main changes and innovations in our approach to the study of small models is the fact that we began to use the formula definability of special subsets of the semantic model of the Jonsson theory under consideration. This was expressed in the introduction of the concept of the Jonsson set and its important special case, as a theoretical set.

The theoretical and practical value of the work. The results obtained are of a theoretical nature and can be used in further research in the field of Universal Algebra and Model Theory to describe countable models of the corresponding classes of theories and algebras, as well as when teaching special courses at university mathematics departments.

Provisions to be defended. The following main results of the dissertation research are presented for defense:

1. sufficient conditions have been found to obtain implications of all types of atomic sets, the closure of which with the help of the closure operator on definable subsets of the semantic model of the considered Jonsson theory gives implications of the corresponding atomic models;
2. a criterion was found for an algebraically prime model for an existentially prime theory when this model is core;
3. the criterion $(\forall 1, \forall 2)$ - cl - Δ -nice a.p of a set was found within the framework of a perfect existentially prime theory;
4. within the framework of a convex perfect existentially prime fragment, a criterion for the core model is obtained;
5. sufficient conditions for the existence of a nice almost-weakly (Σ_1, Σ_1) - cl -atomic and (Σ_1, Σ_1) - cl -algebraically prime model of Jonsson theory and a criterion for their equivalence are found.

Reliability and validity of the conducted research is ensured by the constructiveness of the methods used. General statements are formulated as theorems and their proofs are presented.

Approbation of the work. The main results of the dissertation were reported and discussed at the following seminars and conferences:

- European Summer Meeting of the Association for Symbolic Logic “Logic Colloquium 2019” (Czech Republic, Prague, August 11–16, 2019);
- European Summer Meeting of the Association for Symbolic Logic “Logic Colloquium 2020” (Poland, Poznan, July 19 – 24, 2021);
- European Summer Meeting of the Association for Symbolic Logic “Logic Colloquium 2023” (Italy, Milan, June 5 - 9, 2023);
- Traditional international April mathematical conference (April 2019, IMMM MNVO RK, Almaty, Republic of Kazakhstan);
- 16th Asian Logic Conference (17-21 June 2019, Nur-Sultan, Republic of Kazakhstan);
- international scientific conference “Theoretical and applied issues of mathematics, mechanics and computer science”, dedicated to the 70th anniversary of Doctor of Physical and Mathematical Sciences, Professor M.I. Ramazanov (Kazakhstan, Karaganda, June 12-13, 2019);

- Maltsev readings: international conference (November 16-20, 2020, Novosibirsk, Russia);
- Maltsev readings: international conference (November 20-24, 2021, Novosibirsk, Russia);
- traditional international April mathematical conference in honor of the Day of Science Workers of the Republic (Kazakhstan, Almaty, April 3-5, 2022);
- International scientific and practical conference “Taiman Readings – 2022”, dedicated to the 105th anniversary of Doctor of Physical and Mathematical Sciences, Academician A.D. Taimanov and the 90th anniversary of West Kazakhstan University named after. M. Utemisov (November 30, 2022, Uralsk, Republic of Kazakhstan);
- VII World Congress of Mathematicians of the Turkic World (TWMS Congress-2023) (Kazakhstan, Turkestan, September 20-23, 2023);
- The reports were made in the “Mathematical Logic” laboratory at a seminar at the Institute of Applied Mathematics of Karaganda Buketov University.

The publication of the results. The main results of the dissertation were published in 18 works: 1 article in journal included in the Web Of Science list, 5 articles were published in journal recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan and 12 works in the materials of international scientific conferences. In the works performed with co-authors, the contribution of each of the co-authors is equal.

The structure and scope of the dissertation. The dissertation of 80 pages consists of the following structural elements: introduction, three sections, conclusion, list of sources used.

The first section outlines the basic concepts and theorems of classical Model Theory, the second section describes a result related to R.L. Vaught’s classical problem of countably prime models of complete theories, but in a more general formulation of the concept of countable atomicity. Where the main point is that it focuses on the syntactic properties of special subsets of a fragment of the semantic model of a particular Jonsson theory, in the third part a criterion for a special case of algebraic primeness is found, where the concept of atomicity of the model is obtained by using some formulaic subset of the semantic model of the considered Jonsson theory, and the closure of this set is an existentially closed submodel of this the semantic model.

The dissertation work was performed at the Department of Algebra, Mathematical Logic and Geometry named after professor T.G. Mustafin of the Karaganda Buketov University.

The number of sources used is 54.

Keywords. Jonsson theory, semantic model, prime model, atomic model, algebraically prime model, core model, existentially closed theory, theoretical set, closure operator, pregeometry.