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SOLUTION OF NONLOCAL BOUNDARY VALUE PROBLEMS FOR THIRD AND FOURTH ORDER PARTIAL DIFFERENTIAL EQUATIONS

ANNOTATION
of the dissertation for the degree of Doctor of Philosophy(PhD)
at education program 8D05401- Mathematics

The relevance of the topic.

In the dissertation, nonlocal boundary value problems for partial differential equations of the third and fourth orders are investigated.

At present, the solvability of nonlocal boundary value problems for partial differential equations is being intensively studied. This is connected with the study of many processes in natural science, and this, in turn, leads to a refinement of the mathematical models of the processes under consideration. A number of works on the study of non-local boundary value problems are considered for second-order equations. However, mathematical models of some physical processes and phenomena are based on equations of a higher order.

Third-order partial differential equations underlie many mathematical models that study the dynamics of soil moisture and groundwater, the propagation of acoustic waves in a weakly inhomogeneous medium. In addition, hyperbolic equations with two independent variables of the third and higher order are used as mathematical models of various processes, namely, a non-stationary rectilinear flow of an incompressible fluid of the second order, Navier-Stokes-Oldroyd fluid flows, vibrations of an elastic-viscous string, relaxation of the simplest type, and oscillations rod during aftereffect, the phenomenon of swinging of a cantilever wing, etc.

Recently, boundary value problems for third-order partial differential equations have been considered comprehensively. This is closely related to its practical application. In the works of such authors as Ashiraliev A., Agges N., Hesenchi F., Latrous K., Memou A., Trusdell S., Zhuraev A.K., Sevastyanov V.A., Kozhanov A.I., Kurant R ., Zikirov O.S., Duan K., Lin H., Yuldashev T.K., Apakov Yu.P. and other equations with two independent variables of the third and higher orders were studied as mathematical models of various processes. Dzhumabaev D.S., Assanova A.T., Orumbaeva N.T. consider boundary value problems for second-order hyperbolic equations in their papers. In addition, in the works of Muratbekov M.B., Ospanov M.N., Keldibekova A.B., Kabdrakhova S.S. boundary value problems were studied in various areas for differential equations in partial derivatives of the third order. Works by Tokmurzin Zh.S. were devoted to initial-boundary value problems for differential equations of the fourth order.

In this paper, there are considered initial-boundary value problems with two independent variables for partial differential equations of the third and fourth orders with various non-local conditions. To solve them, new functions were introduced and initial-boundary value problems related to the family of equations with decreasing

order were obtained. For some problems, the parameterization method of D.S. Dzhumabaev was applied. Algorithms for finding approximate solutions to these problems were constructed, conditions for the convergence of the constructed algorithm were obtained, and the existence and uniqueness of a solution to the problem were proved.

The goal of the work. Obtain conditions for the solvability of non-local boundary value problems for differential equations in partial derivatives of the third and fourth orders, construct constructive algorithms for finding their solutions.

Research objectives:

1. To construct algorithms for finding solutions to initial-boundary value problems for third-order partial differential equations and obtain conditions for their convergence;

2. To establish coefficient signs for the unique solvability of a boundary value problem with nonlocal conditions for differential equations in partial derivatives of the third order with a mixed derivative;

3. To obtain sufficient conditions for the unique solvability of a two-point initial-boundary value problem for third-order pseudoparabolic equations;

4. To apply the method of parametrization of the solution of the initial-boundary value problem for differential equations in partial derivatives of the third order;

5. To investigate the unique solvability of a nonlocal initial-boundary value problem for fourth-order partial differential equations.

Object of the research: nonlocal boundary value problems for differential equations in partial derivatives of the third and fourth orders.

Subject of the research: Initial-boundary value problems for partial differential equations of the third and fourth orders with various non-local conditions, algorithms for finding their solutions, convergence conditions for the constructed algorithms, uniqueness of the solution.

Research methodology.

There were used the methods of functional analysis and the method of parametrization in the work.

Scientific novelty.

Nonlocal boundary value problems for third and fourth order partial differential equations were investigated and the following results were obtained:

1. Algorithms for finding solutions to initial-boundary value problems for third-order partial differential equations and conditions for their convergence;

2. Coefficient signs for the unique solvability of a boundary value problem with nonlocal conditions for differential equations in partial derivatives of the third order with a mixed derivative;

3. Sufficient conditions for the unique solvability of a two-point initial-boundary value problem for third-order pseudoparabolic equations;

4. Method of parametrization of the solution of the initial-boundary value problem for differential equations in partial derivatives of the third order;

5. The unique solvability of a nonlocal initial-boundary value problem for fourth-order partial differential equations.

Theoretical and practical value of the work.

The results obtained in the course of the work are of a theoretical nature and can be used in the construction of algorithms for solving nonlocal boundary value problems for partial differential equations of the third and fourth orders, as well as when reading special courses in mathematics in universities.

Provisions submitted for presentation.

The following are submitted for presentation:

1. Algorithms for finding solutions to initial-boundary value problems for third-order partial differential equations and conditions for their convergence;

2. Coefficient signs for the unique solvability of a boundary value problem with nonlocal conditions for differential equations in partial derivatives of the third order with a mixed derivative;

3. Sufficient conditions for the unique solvability of a two-point initial-boundary value problem for third-order pseudoparabolic equations;

4. Method of parametrization of the solution of the initial-boundary value problem for differential equations in partial derivatives of the third order;

5. The unique solvability of a nonlocal initial-boundary value problem for fourth-order partial differential equations.

The credibility and validity of the conducted researches are ensured by the constructiveness of the methods used. General statements are formulated in the form of theorems and their proofs are presented, auxiliary statements are formulated in the form of lemmas and they are proved.

Approbation of the work.

The main results of the dissertation were reported and discussed at the following conferences and seminars:

- International scientific and practical conference dedicated to the 30th anniversary of the independence of the Republic of Kazakhstan and the 20th anniversary of the Kazakhstan branch of Moscow State University. M.V. Lomonosov "Problems of modern fundamental and applied mathematics" (June 4, 2021, Nur-Sultan, Republic of Kazakhstan);

- VI International Conference "Nonlocal Boundary Value Problems and Related Problems of Mathematical Biology, Informatics and Physics" (December 5-9, 2021, IAMA KBSC RAS, Nalchik, RF)

- IX International Conference (May 24-28, 2022, Aktobe Regional University named after K. Zhubanov, Aktobe, Republic of Kazakhstan);

- International Scientific Conference "Actual Problems of Mathematics, Mechanics and Informatics", dedicated to the 80th anniversary of Professor T.G. Mustafin (September 8-9, 2022, Karaganda University named after Academician E.A. Buketov, Karaganda, Republic of Kazakhstan);

- International Conference "Dynamical Systems, Modeling, and Mathematical Sciences" (September 23-25, 2022, Dubai, UAE);

- International scientific conference "Non-classical equations of mathematical physics and their applications" (October 6-8, 2022, National University of Uzbekistan named after Mirzo Ulugbek, Tashkent, Republic of Uzbekistan);

- Traditional international April mathematical conference (April 5-7, 2023,

IMMM MNVO RK, Almaty, Republic of Kazakhstan);

- International scientific and practical conference "Trends in the development of modern mathematics and its teaching in the context of digitalization of education" (April 27-28, 2023, Peoples' Friendship University named after academician A. Kuatbekov, Shymken, Republic of Kazakhstan);

- Seminar of the Institute of Applied Mathematics of the Karaganda University named after Academician E.A. Buketova.

Publications. The main results of the dissertation were published in 12 works: 2 articles in journals included in the Scopus list, 2 articles were published in journal recommended by the Committee for Quality Assurance in the Field of Science and Higher Education of the Ministry of Science and Higher Education of the Republic of Kazakhstan and 8 works in the materials of international scientific conferences.

In the works performed with co-authors, the contribution of each of the co-authors is equal.

The structure and scope of the dissertation.

The dissertation work with a scope of 101 pages consists of the following structural elements: an introduction, two sections, a conclusion, a list of sources used.

In the first section, initial-boundary value problems for third-order partial differential equations with various nonlocal conditions were studied, algorithms for finding solutions to these problems were constructed, conditions for the convergence of the constructed algorithms were obtained, and were proved the uniqueness of the solution. In the second section, were studied a nonlocal initial-boundary value problem for fourth-order differential equations. To solve this problem, the parameterization method of D.S. Dzhumabaev was applied. Sufficient conditions are obtained for the unique solvability of the problem under consideration with respect to the initial data.

Examples are given.

The number of sources used is 85.

Keywords. Partial differential equations, nonlocal boundary value problems, third-order pseudoparabolic equation, fourth-order partial differential equation, solvability conditions, algorithm, approximate solution.