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SOME INTEGRABLE SPIN SYSTEMS AND RELATED NONLINEAR EQUATIONS

ANNOTATION
Of dissertation for the degree of Doctor of Philosophy (PhD)
in specialty 6D060100-«Mathematics»

Relevance of the research topic. The modern theory of non-linear evolutionary equations has become a significant area of fundamental science with its own problems, principles and methods of solution. One of these problems is certainly the generalization of methods for obtaining exact solutions to nonlinear evolutionary equations, as well as the mathematical modeling of new spin systems, which are equivalent analogues of some classical nonlinear integrable systems.

In recent decades, nonlinear phenomena in magnetic media have been actively studied. This interest is due to the fact that magnetic crystals are widely used in many areas, for example, when using the properties of electromagnetic oscillations and waves in electrical engineering, microwave technology and nanotechnology. Studies of such physical processes require mathematical modeling. The equations describing the phenomena in magnets are related to spin systems, which are also non-linear equations. One of the first representatives of spin systems is the Heisenberg ferromagnet equation. For this reason, in many modern works on nonlinear theory, spin systems are also called the generalized Heisenberg ferromagnet equations.

The study of nonlinear spin systems is closely related to the concept of equivalence. In nonlinear theory, there are gauge and geometric equivalences.

The concept of gauge equivalence first arose in the work of V.E. Zakharova and L.A. Takhtajyan. They noted that the nonlinear Schrödinger equation in the case of attraction by the inverse scattering method is equivalent to a continuous isotropic chain of Heisenberg spins.

The emergence of gauge equivalence gave birth to another independently developing branch of integrable nonlinear equations. The gauge transformation is applicable to equations that can be solved using the inverse scattering method. The meaning of gauge equivalence is that any Lax pair of a nonlinear integrable equation can be reduced to a Lax pair of a spin system in the reverse order. Thus, it is established that each solution of the first corresponds to the solution of the second.

In parallel, a geometric approach to studying nonlinear integrable equations was developed. With the advent of the work of H. Hashimoto, significant development of the geometric system took place. Hashimoto discovered that the internal equation describing the curvature and torsion of a thin vortex filament reduces to a non-linear Schrödinger equation in three-dimensional Euclidean space. Using the Hashimoto transformation, M. Lakshmanan and others derived the Heisenberg spin chain equation. Thus, geometric equivalence between the nonlinear Schrödinger equation and the Heisenberg ferromagnet equation was established for the first time. The application of differential geometry has become a separate branch of the theory of nonlinear equations.

Thus, the combination of differential-geometric and gauge methods allows us to solve several important problems and understand many issues, some of which are the content of the dissertation. It turned out that the establishment of an equivalent relationship helps to systematize and classify the known nonlinear systems. Also, to come to a conclusion about the properties of the new system knowing the properties of its equivalent analog, moreover, to show the connection between their solutions. All of the above determines the relevance of the proposed research.

The aim of the work. The aim of this paper is to study the relationship between integrable spin systems and nonlinear evolution equations by establishing gauge and geometric equivalences. In addition, the representation of three formulations of the nonlinear Schrödinger equation with respect to the hybrid frame in three-dimensional Minkowski space and finding their respective solutions.

The objects of research are nonlinear integrable spin systems and related nonlinear integrable equations.

The subject of research is the establishment of an equivalent connection between nonlinear integrable spin systems and nonlinear integrable equations.

Scientific provisions submitted for defense:

The following research results, obtained for the first time in the framework of the dissertation work, are presented for defense:

1) Three formulations of the nonlinear Schrödinger-type equation with respect to the hybrid frame in the three-dimensional Minkowski space are obtained. Solutions of a nonlinear Schrödinger-type equation are obtained for three formulations.

2) A spin system is derived, which is a generalized equation of a Heisenberg ferromagnet, associated with an integrable nonlinear Hunter-Saxon equation.

3) A connection is established in the form of gauge and geometric equivalence between the generalized Heisenberg ferromagnet equation and the Hunter-Saxon equation. The connection between the solutions of the generalized Heisenberg ferromagnet equation and the Hunter-Saxon equation is determined.

4) Gauge equivalent analogs of integrable equations of the Yajima-Oikawa type are found, which are generalized equations of the Heisenberg ferromagnet with self-consistent potentials. The corresponding Lax representations of these systems are given. The generalized equations of the Heisenberg ferromagnet with self-consistent potentials arise from the condition of compatibility of matrices $U(x,t)$, $V(x,t)$, dimension 2×2, 3×3.

Research methods. The dissertation work uses recognized methods of the theory of integrable nonlinear equations. In particular, gauge and geometric equivalences are used to determine the connection between spin systems and integrable nonlinear equations. This helps to understand the nature of spin systems and related non-linear evolution equations. Spatial curves of integrable nonlinear equations are constructed using methods of differential geometry. Moreover, three formulations are presented related to the non-linear Schrödinger-type equation in the directions of *h*-lines and r-lines with respect to the hybrid frame in the three-dimensional Minkowski space.

Scientific novelty of the research. In this work, the following new results are obtained:

1 Solutions of the modified nonlinear Schrödinger equation in the directions of h-lines and r-lines with respect to the hybrid frame in the three-dimensional Minkowski space are obtained.

2 Geometrical equivalence between the generalized Heisenberg ferromagnet equation and the Hunter-Saxon equation has been established.

3 The gauge equivalence of the generalized Heisenberg ferromagnet equation and the Hunter-Saxon equation is proved. The connection between their solutions is determined.

4 Spin systems are obtained which are gauge equivalent analogues of Yajima-Oikawa type equations.

Theoretical and practical significance of the research.
The results of the work are theoretical in nature and can be used in the theory of integrable nonlinear equations, in mathematical physics and in differential geometry. Also, the results of the dissertation work can be applied in the educational process for reading elective courses for bachelors, masters and doctoral students of the specialty "Mathematics".

Communication of dissertation work with other research works.
The dissertation work was carried out in accordance with the plans of research works of the following projects of grant funding of the Ministry of Education and Science of the Republic of Kazakhstan:

1. AP08856912 «Research of the geometry of integrable dispersion and dispersionless equations» 2020-2022 y.

2. AP14971227 «Investigation of some integrable generalized Heisenberg ferromagnet equations» 2022-2024 y.

Approbation of the research results. The results obtained in the dissertation work were reported and discussed at international and republican conferences:

1. VI Congress of the Turkic world mathematical society (TWMS 2017) (Astana, 2017).

2. Geometric Methods in Physics XXXVII (Bialowieza, 2018).

3. Lomonosov-2019: 15th international scientific conference of students, master's students and young scientists "Lomonosov - 2019" (Nur-Sultan, 2019).

4. International conference "Actual problems of analysis, differential equations and algebra" (EMJ-2019) (Nur-Sultan, 2019).

5. Lomonosov-2020: 16th international scientific conference of students, master's students and young scientists "Lomonosov - 2020" (Nur-Sultan, 2020).

6. International Scientific Conference "Ufa Mathematical School-2021") (Ufa, 2021).

In addition, the results obtained were reported and discussed at scientific seminars of the Department of "Fundamental Mathematics" of the L.N. Gumilyov.

The reliability and validity of the scientific provisions, conclusions and results of the dissertation is confirmed by the publication of the obtained results in journals with a non-zero impact factor.

Publications. Based on the results of the dissertation work published:

Articles indexed in the Web of Science database or in the Scopus database

1. N. E. Gurbuz, R. Myrzakulov, Zh. Myrzakulova. Three anholonomy densities for three formulations with anholonomic coordinates with hybrid frame in R_1^3 // Optik. – 2022. – Vol. 261, P. 169161 (WoS IF - 2.840, Scopus Q2, процентиль – 67 %, Citescore - 4.8).

2. Zh. Myrzakulova, G. Nugmanova, K. Yesmakhanova, N. Serikbayev, R. Myrzakulov. Integrable generalized Heisenberg ferromagnet equations with self-consistent potentials and related Yajima-Oikawa type equations // Ufa Mathematical Journal. . – 2023. – Vol. 15. No 1 P. 122-134. (Scopus Q3, процентиль – 34 %)

Articles in domestic or foreign scientific publications recommended by KOKSNVO:

1. Мырзакулова Ж.Р., Есмаханова К.Р. Бездисперсионный предел уравнения Ma // Вестник КазНУ им. аль-Фараби. - №2 (102). – С. 12-21. (2019)

2. Zh. Myrzakulova, S.R. Myrzakul. Gauge equivalence between the Γ-spin system and (2+1)-dimensional two-component nonlinear Schrodinger equation // News NAS RK. -Vol.2, N330, 112 – 119. (2020)

3. Zh. Myrzakulov, K. Yesmakhanova, Zh. S. Zhubayeva. Equivalence of the Hunter-Saxon equation and the generalized Heisenberg ferromagnet equation // News NAS RK. -Vol.2, N336, 33 – 38. (2021)

The scope and structure of the dissertation. The dissertation work consists of an introduction, 4 sections, a conclusion and a list of references from 119 titles, contains 102 pages of the main computer text.

The main content of the dissertation. The work consists of five sections.

The introduction reflects the rationale for the relevance of the topic of the dissertation work, the purpose, object, subject, research objectives, the methods used, the rationale for the scientific novelty of the work, its theoretical and practical significance, the scientific positions submitted for defense, the number of available publications, information about the approbation of the work and the degree of its development.

The first section introduces definitions and concepts related to nonlinear integrable equations and differential geometry. In the second chapter, we study nonlinear Schrödinger type equations in three-dimensional Minkowski space with respect to a hybrid frame. The third section is devoted to the study of the relationship between the generalized Heisenberg ferromagnet equation and the Hunter-Saxon equation. In the fourth section, equivalent gauge analogs of Yajima-Oikawa type equations are found.

In the final section of the work, the main results obtained during the period of the dissertation are presented and summarized.