

## ABSTRACT

**of the dissertation work of Bayandiyev Yerik Nurlanovich for the degree of Doctor of Philosophy (PhD) in the specialty "6D060100-Mathematics" on the topic «On the properties of solutions of singular hyperbolic equations with rapidly increasing coefficients»**

**The relevance of the topic.** The dissertation work is devoted to the study of the invertibility of differential operators of hyperbolic type given in an unbounded domain with singular coefficients, the continuity of the resolvent, compactness and other spectral properties.

The theory of differential equations of hyperbolic type is one of the most important branches of the theory of differential equations with independent derivatives and attracts the attention of scientists with its wide application possibilities. This is due to the theoretical significance of the results obtained, as well as their applications in radiophysics and electrical engineering, magnetohydrodynamics, and other areas of natural science.

Existence, uniqueness and qualitative properties of solutions of differential equations of hyperbolic type are quite fully considered in the works of J. Hadamard, K. Friedrichs, S. L. Sobolev, L. Garding, O. A. Ladyzhenskaya, A. V. Bitsadze, J. Leray, A. Ya. Nakhushiev, T. Sh. Kalmenov, T. Ya. Kiguradze, A. V. Filinovskii and others.

These papers consider problems with Darboux and Goursat periods, the Cauchy problem, and other problems for equations with constant or limited coefficients.

M. Nagumo studied the solution of equations of hyperbolic type with continuous and bounded coefficients in  $R^n$ -space.

The following problems are important for differential operators of hyperbolic type, defined in an infinite domain and whose coefficients grow infinitely:

- the existence of a resolvent (inverse operator) of the operator;
- separability of the operator in the functional space under consideration;
- compactness of the resolvent;
- evaluation of singular values (s-numbers) of the resolvent;
- estimate of the eigenvalues of the resolvent.

The main difference between the methods for studying hyperbolic differential operators and elliptic differential operators is that they do not find energy spaces. For this reason, new difficulties arise in the way of elucidating the smoothness properties of functions related to the domain of definition of a hyperbolic differential operator.

In conclusion, we note that the solution of these problems for one class of singular differential operators of hyperbolic type is very important both in theoretical and practical terms, is an urgent problem.

Therefore, in this paper, we study the above problems for a differential operator of hyperbolic type.

**The purpose of the work.** The existence of a resolvent, compactness, separability, and spectral properties of differential operators of hyperbolic type with infinitely increasing coefficients in an infinite domain are studied in this paper.

**Research objectives:**

- find conditions for the existence of a resolvent of a differential operator of hyperbolic type with infinitely increasing coefficients in an infinite domain;
- find separability conditions for a differential operator of hyperbolic type with infinitely increasing coefficients in an infinite domain;
- find necessary and sufficient conditions that testify to the compactness of the resolvent of a differential operator of hyperbolic type with infinitely increasing coefficients in an infinite domain;
- to investigate the spectral properties of the above differential operator of hyperbolic type with coefficients increasing indefinitely in an infinite domain.

**The objects of research.** Conditions for the existence of an inverse operator, smoothness (separability) of functions in a given domain, compactness and spectral properties of an inverse operator for one class of differential operators of second-order hyperbolic type.

**Research methods.** In the dissertation work, the unit decomposition method, the localization method, the method of coercive estimates, partitioning of coefficients, Fourier transforms, spectral theory of linear operators, embedding theorems in weighted functional spaces, Kolmogorov widths were used.

**Scientific novelty.** The following new scientific results were obtained in the work:

- a) for one class of hyperbolic operator defined on a strip:
  - conditions for the existence of an inverse operator are found;
  - separability conditions for a differential operator of hyperbolic type are obtained;
  - necessary and sufficient conditions are found that testify to the compactness of the resolvent of a differential operator of hyperbolic type;
  - a two-sided estimate for the singular values (s-numbers) of the resolvent of a differential operator of hyperbolic type is obtained.

The resulting estimates allow us to approximate the resolvent by finite-dimensional operators.

-using the inequalities taken for singular values, an estimate for the eigenvalues of the operator is found.

b) In work, also proven:

- the existence of an inverse operator for a class of hyperbolic differential operators with infinitely growing coefficients;
- a condition is found that ensures the separability of a differential operator of hyperbolic type in the case of an unbounded domain with a strongly growing coefficient.

**Theoretical and practical value of the work.**

The results obtained are predominantly theoretical. In general, they complement the theory of partial differential equations and operators with unbounded coefficients. They can be used in the spectral theory of differential operators and in the study of qualitative properties of singular differential equations.

The scientific results found can be used by students, undergraduates and doctoral students in special courses on the equations of differential and mathematical physics.

**Approbation of the work.** The main results of the work were presented in the following conferences:

1. International conference "Theoretical and applied problems of mathematics, mechanics and informatics". Karaganda, 2019..

2. International conference " Actual problems of analysis, differential equations and algebra ". Astana, 2019.

3. International conference "Actual problems of the theory of optimal control, dynamical systems and operator equations". B.N. Yeltsin Kyrgyz-Russian Slavic University. Bishkek. 2022.

4. XIV International Scientific and Practical Conference "Development, education, culture: integration trends in the modern world". Oslo, Norway, 2023.

Also, the individual results of the work were discussed in the following seminars:

- Scientific seminar "Functional analysis and its application" (headed by academicians of the National Academy of Sciences of the Republic of Kazakhstan M. Otelbaev and R. Oinarov, professors E.D. Nursultanov, K.N. Ospanov).

- Scientific seminar "Spectral theory of differential operators" under the guidance of Professor of the Department of Mathematics in Education, Taraz Regional University named after M.Kh.Dulaty.

**Publications.** The main results of the dissertation have been published in 8 scientific articles and conference materials, including 3 articles in publications recommended by the authorized body, 1 article in a rating publication included in the Scopus database, 1 article in foreign publications.

**The structure and scope of the dissertation.** The dissertation consists of an introduction, three parts (each part is divided into paragraphs), a conclusion and a list of references.

**Keywords.** Differential operator of hyperbolic type, resolvent, operator separability, singular numbers.