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BOUNDARY VALUE PROBLEMS FOR A TWO-DIMENSIONAL HEAT EQUATION ON A SPATIAL VARIABLE IN A CONE

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100– Mathematics

The relevance of the topic.

Boundary value problems of heat conduction in degenerating domains arise in the study of thermal processes in the intercontact space. In modern electrical devices, super-strong and ultra-weak currents are very often used, so there is a need to study new phenomena that have not been previously observed at normal, medium-range currents. For example, it has been experimentally established that when the contacts of current switches are opened for a short time, a liquid metal bridge appears, that significantly affects the erosion of the contact material. In the work of S.N. Kharin, a mathematical model is presented that describes the transient phenomena accompanying a short vacuum arc at the initial stage of opening contacts.This allows the authors to describe the evolution of the transient short anode dominant arc, which occurs immediately after the destruction of the molten bridge. From a mathematical point of view, the features of the problem under consideration consist, firstly, in the presence of a time-varying boundary of the domain of definition, the boundary change depends on conditions contact opening. Secondly, the solution domain degenerates into a point at the initial moment of time, since at the initial moment of time the contacts are in a closed state.

The fundamental difference between boundary value problems for parabolic equations in evolving domains and classical problems (for cylindrical domains) is that the methods of separation of variables and integral transformations are not applicable to such problems, since it is impossible to coordinate the solution of the equation with the boundary movement of the heat transfer domain.The application of the method of thermal potentials makes it possible to reduce the boundary value problem with a moving boundary to a Volterra type integral equation of the second kind. If the definition domain does not degenerate into a point at the initial moment of time, then the obtained equation can be solved by the method of successive approximations, i.e. the integral equation, and with it the boundary value problem, have a unique solution. If the definition domain degenerates into a point at the initial moment of time, then an integral equation of the Volterra type becomes special (singular), since the corresponding homogeneous equation, and hence the original homogeneous boundary value problem, may have non-zero solutions. Moreover, Picard's method is not applicable to the Volterra integral equation.

The peculiarity of the studied problem in this paper is the degeneration of the solution domain into a point at the initial moment of time and the need to study the problem for sufficiently small time values.

The paper investigates a two-dimensional boundary value problem on spatial variables in an inverted cone

$$G = \{(x; y, t): x^2 + y^2 < t^2, 0 < t < T\}$$

for the equation

$$\frac{\partial u(x,y,t)}{\partial t} = a^2 \left(\frac{\partial^2 u(x,y,t)}{\partial x^2} + \frac{\partial^2 u(x,y,t)}{\partial y^2} \right) \quad (1)$$

with a condition on a lateral surface of the cone

$$u(x,y,t) = u_c(x,y,t), \sqrt{x^2+y^2} = t, 0 < t < T, \quad (2)$$

where $u_c(x,y,t)$ is a given function.

Under certain physical and technical assumptions, the boundary value problem (1)-(2) simulates the temperature field in the plasma body of an electric discharge between high–voltage contacts that were initially in a closed state. Since there are no devices that could measure the specified temperature field due to the short duration of the process, it is necessary, at least qualitatively, to assess the nature of the ongoing thermal processes using mathematical modeling methods.

To study the problem (1)–(2), thermal potentials are constructed. The solution to the problem under study is presented as a sum of the constructed potentials, and the problem is reduced to a singular integral equation of the Volterra type of the second kind, namely, to the degenerate Abel equation, which can be considered as an integral equation of the third kind

$$t\,\psi(t) - \frac{\lambda}{\sqrt{\pi}} \int_0^t \frac{\psi(\tau) d\,\tau}{\sqrt{t-\tau}} = F(t), 0 < t < T < \infty, \quad (3)$$

where λ is a given positive quantity, and $\{F(t), t \in (0,T)\}$ is a given function.

The order of degeneracy in the equation (3) under study is equal to one, the kernel of the integral operator has a weak singularity and defines the Abel integral operator. We study the solvability of equation (3) in the weight class of essentially bounded functions.

The purpose of the work.

Formulation of boundary value problems for heat conduction equations in domains degenerating into a point at the initial moment of time, and investigation of their solvability in various functional spaces; study of related Volterra integral equations of the second kind.

Research objectives:

1. Give a statement of boundary value problems for the heat equation in non-cylindrical domains and describe the spaces of solutions and given functions;
2. Construct thermal potentials to represent the solution of the boundary value problem;
3. Reduce the boundary value problem to the degenerate Abel equation;
4. Determine the uniqueness classes for the boundary value problems under study in various functional spaces.

5. To investigate for solvability in Sobolev Hilbert spaces the boundary value problem of heat conduction, two-dimensional in spatial variables, in a degenerating domain.

The object of the study: boundary value problems for parabolic type equations in domains degenerating into a point at the initial moment of time.

Subject of research: solvability of boundary value problems for heat conduction equations in domains degenerating into a point at the initial moment of time in Lebesgue and Sobolev spaces.

Research methods.

The methods of the general theory of differential equations, functional and complex analysis, the method of reducing boundary value problems to integral equations, the method of a priori estimates are used in the work.

Scientific novelty.

The paper proposes new formulations of boundary value problems for the heat equation in functional classes. The feature of the problems under consideration is that there is a need to study the solvability of integral equations of the third kind, namely the degenerate Abel equation. A technique is also proposed to study the problem solvability in Sobolev spaces in domains degenerating into a point at the initial moment of time.

Theoretical and practical value of the work.

The results of the dissertation are theoretical in nature. It has developed a technique for studying a number of boundary value problems for two-dimensional spatial variable heat conduction equations based on reducing the studied problems to Volterra integral equations of the second kind.

In addition, a method for studying boundary value problems in degenerate domains has been developed, based on the representation of the original degenerating domain as a system of nested non-degenerating domains, the union of which in the limit gives the definition domain (cone) of the original problem solution.

The main provisions for defense.

The following provisions are submitted for defense:

1^0Statements of boundary value problems for the heat equation in functional classes with description of solution spaces and input data of problems;

2^0Construction of thermal potentials to represent the solution of the problem;

3^0Reduction of the boundary value problem to the degenerate Abel equation

4^0Proof of solvability of the studied boundary value problems in various functional spaces.

Reliability and validity.

The reliability and validity of the studies are ensured by the constructiveness of the developed and used methods..The auxiliary statements of the problematic issues of each section are formulated in the form of lemmas and statements, and they are strictly proved, and the general ones are in the form of theorems and their proofs are presented in a detailed presentation.

Approbation of the work.

According to the results of the dissertation, reports were made at international conferences and at conferences abroad:

The Fourth International Conference on Analysis and Applied Mathematics, 6-9 June 2018, Lefkosha (Nicosia), Mersin 10, Turkey;

"Problems of modeling processes in electrical contacts" International Mathematical Conference dedicated to them. The Day of Science and the 80th anniversary. anniversary of the Academy. NAS RK S.N.Kharina, Almaty, 2019;

"Marchuk Scientific Readings - 2020" international conference dedicated to the 95th anniversary of the birth of Academician G.I. Marchuk, Novosibirsk, 2020, etc.;

at the seminar under the guidance of Prof. M.T. Genaliev (Imim, Almaty);

at the seminar under the guidance of Prof. Ramazanova M.I. (KarU named after Academician E.A.Buketov); and others.

Publications.

The main results of the dissertation were published in 19 papers: 1 article - in the journal included in the Scopus list, 9 articles were published in journals recommended by the Committee for Quality Assurance in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan and 9 papers - in the materials of international scientific conferences.

In the works performed with co-authors, the contribution of each of the co-authors is equal.

The structure and scope of the dissertation.

Dissertation work of 128 pages, consists of the following structural elements: designations and abbreviations, introduction, three sections, conclusion, list of sources used.

The number of sources used is–90.

Keywords. thermal potentials, Volterra type equations of the second kind, Abel equations, Sobolev spaces, boundary value problem of thermal conductivity.