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QUANTUM CALCULUS AND ITS APPLICATION TO FRACTIONAL DIFFERENTIAL EQUATIONS

ABSTRACT

Dissertations for the degree of Doctor of Philosophy (PhD) in the educational program "8D05401-Mathematics"

The relevance of the topic. Recently, the general theory of boundary value problems for differential equations of odd order with partial derivatives has been extensively developed, the reasons for such development of which are: differential equations of odd order are often used in hydromechanics, electrodynamics, the theory of solids and waves of physics.

One of the representatives of partial differential equations of odd order is the differential equations of the third.

Equations in partial derivatives of the third order underlie mathematical models of many phenomena and processes. In particular, this group includes the nonlinear Korteweg-de Vries equation, which is the basic equation of modern mathematical physics. This equation is first encountered in the works of J. Boussinesky. But the final studies were carried out in 1895. D. Korteweg and G. de Vries. Further studies of this equation were carried out by many famous scientists. The latest results and full descriptions are provided in the works of R. Temam, T. Kato, J.Y. Bona, R. Smith and J. Whitham.

In most papers, to study solutions of nonlinear equations, one first studies its linearized form. The main feature of this method, unlike other methods, in our case is the following approach: first, we prove questions about the existence of an inverse operator, the smoothness of elements from the domain of the operator, and the compactness of the set associated with the domain of the linearized operator. Then, these obtained results are further used to study the properties of solutions of nonlinear differential equations.

The study of partial differential equations of odd order, including the linearized form of the Korteweg-de Vries equations, are considered in the works of R. Temam, T. Kato, J.Y. Bona, R. Smith, Whitham, A. Villanueva, E. Taflin, Y. Turbal, Chunxiong Zheng and other scientists. In these papers, various boundary value problems were considered for differential equations whose coefficients are constant or continuous and whose growth is limited.

However, in a number of cases, when using differential equations of odd order, one has to consider problems in an unbounded domain. For example, in electrodynamics, quantum physics, phenomena that describe the propagation of waves, it is necessary to know the properties of solutions to differential equations, in which odd-order coefficients increase indefinitely. Therefore, for linear differential operators of odd order in the case of an unbounded domain with strongly growing coefficients, the following problems are important:

- operator closure;
- the existence of a resolvent;
- maximum smoothness of functions from the domain of definition of the linearized Korteweg-de Vries operator (separability of the operator);
- compactness of the resolvent of the linearized Korteweg-de Vries operator;
- spectral properties of a linear differential operator of the Korteweg-de Vries type, i.e. estimates of singular and eigenvalues.

Methods for studying differential operators of odd order have their own peculiarities in comparison with differential operators of even order. For example, differential operators of odd order in the space L_2 are not semi-restricted, i.e., the condition is not satisfied: $< Lu, u > \geq \gamma \cdot \|u\|_2^2$, where L is a differential operator, $< \cdot, \cdot >$ is an inner product in the space L_2, $\|\cdot\|_2$ is the norm, γ is any final real number. Failure to comply with this condition makes it difficult to study the differential properties of functions from the domain of a differential operator of odd order.

Therefore, we note that the solution of the above problems for differential operators of odd order is very important from a scientific point of view.

Looking through the literature, we see that the existence, separability, spectral and approximative properties of the resolvent of the linearized Koteweg-de Vries differential operator, whose coefficients grow infinitely in an unbounded region, have not been studied enough. Therefore, it seems important and topical to study the above questions in science and practice for a linearized Korteweg-de Vries operator with singular coefficients.

In this work, together with the Korteweg-de Vries operator, we consider a parabolic differential operator, which is often encountered in mathematical physics. The partial time derivative of this operator is of odd order. Therefore, the existence of a resolvent and the separability of this differential operator, whose coefficient increases indefinitely in an unbounded domain, is a very important problem that requires deep investigation.

The purpose of the work. The existence of a resolvent, compactness, separability, and spectral properties of partial differential operators of odd order whose coefficients grow unboundedly in an infinite domain are studied in this paper.

Research objectives:

- To find conditions for the existence of a resolvent of a linearized Koteweg-de Vries differential operator, whose coefficients increase indefinitely in an infinite domain;

- find the conditions for separability of the linearized Koteweg-de Vries differential operator, whose coefficients increase indefinitely in an infinite domain;

- to find the necessary and sufficient conditions that testify to the compactness of the resolvent of the linearized Koteweg-de Vries differential operator, whose coefficients grow unboundedly in an infinite domain;

- to investigate the spectral properties of the above-mentioned third-order differential operator, whose coefficients grow indefinitely in an infinite domain.

The objects of research. Existence and compactness of the resolvent of a linearized singular Korteweg-de Vries differential operator, as well as the existence of a resolvent and the separability and spectral properties of a parabolic differential operator in the case of an unbounded domain with strongly growing coefficients.

Research methods. The dissertation work used the method of localization, the method of a priori estimates, Fourier transforms, linear operator methods, the theory of weighted function spaces, the compactness method.

Scientific novelty. The following new scientific results were obtained in the work:

- For one class of linearized Korteweg de Vries differential operators, whose coefficients grow infinitely at infinity in an infinite domain, the following results were obtained:

a) conditions for the existence of a resolvent for the coefficients are found;

b) separability conditions for the linearized Korteweg-de-Vries operator are obtained;

c) necessary and sufficient conditions are found that testify to the compactness of the resolvent of the linearized Korteweg-de Vries differential operator;

d) a two-way estimation of singular numbers (s-numbers) of the resolvent of the linearized Korteweg de Vries operator is obtained. Consequently, it is possible to evaluate the operator's eigenvalues from above, that is, these found values open a direct path to approximate solutions.

In the work, it is also proved

- the existence of an inverse operator for one class of parabolic differential operators with infinitely increasing coefficients.

- a condition is found that ensures separability of a differential operator of parabolic type in the case of an unbounded domain with a strongly increasing coefficient.

Theoretical and practical value of the work. In this work, the scientific results are theoretical in nature. The scientific conclusions obtained in the work can be used in a deep study of the qualitative properties of the resolvent of differential operators of odd order. In particular, they can be used to study the existence, compactness and spectral properties of the resolvent of odd-order differential operators in an infinite domain.

The found scientific results can be used by students, undergraduates and doctoral students in special courses on differential equations and mathematical physics.

Approbation of the work. The main results of the work were presented in the following conferences:

1. Traditional international scientific April conference dedicated to the Day of Science Workers. Institute of Mathematics and Mathematical Modeling. Almaty, 2018, 2020, 2021.

2. International conference "Theoretical and applied problems of mathematics, mechanics and informatics". Karaganda, 2019.

3. IV International scientific conference "Actual problems of the theory of optimal control, dynamical systems and operator equations". B.N. Yeltsin Kyrgyz-Russian Slavic University. Bishkek. 2022.

Also, the individual results of the work were discussed in the following seminars:

- Scientific seminar "Functional analysis and its application" (headed by academicians of the National Academy of Sciences of the Republic of Kazakhstan M. Otelbaev and R. Oinarov, professors E.D. Nursultanov, K.N. Ospanov).

- Scientific seminar of the department "Basic Mathematics". Nur-Sultan, 2020.

- Scientific seminar "Spectral theory of differential operators" under the guidance of Professor of the Department of Mathematics in Education, Taraz Regional University named after M.Kh.Dulaty.

Publications. The main results of the dissertation have been published in 9 scientific articles and conference materials, including 3 articles in publications recommended by the authorized body, 1 article in a rating publication included in the Scopus database, 1 article in foreign publications.

The structure and scope of the dissertation. The dissertation consists of an introduction, three parts (each part is divided into paragraphs), a conclusion and a list of references.

Keywords. Odd-order differential operator, Korteweg-de Vries equation, resolvent, division operator, singular numbers.