

**ONERBEK ZHOMART MURATULY**

**THE BOUNDEDNESS AND COMPACTNESS OF SOME OPERATORS  
IN MORREY TYPE SPACES WITH VARIABLE EXPONENT**

**ANNOTATION**

**of dissertation for the degree of Doctor of Philosophy (PhD) in  
specialty 6D060100-Mathematics**

**Relevance of the research topic.** Function spaces play an important role in solving various theoretical and applied problems in mathematics. The study of the properties of various function spaces is devoted to the monographs of the well-known S. M. Nikolsky, O. Besov, H. Triebel, V. I. Burenkov. Survey articles by Burenkov et al. are devoted to the theory of function spaces.

In recent decades, Morrey-type spaces have been actively studied. It is known that Morrey-type spaces and their generalizations (the so-called Morrey-type spaces) play an important role in the theory of functional analysis. The classical Morrey spaces and their generalizations arose in connection with certain questions in the theory of differential equations. More recently, Morrey space has been widely used in operator theory. There are many books and review articles on Morrey spaces and their applications.

The maximal Hardy-Littlewood operator in the Lebesgue space and Morrey-type spaces, the fractional maximum Hardy-Littlewood operator, the Rees potential, singular integral operators, etc. — the theory of restrictions of the classical operators of real analysis is well studied. Necessary and sufficient conditions for weight functions are found that ensure boundedness of classical operators in Morrey-type spaces for the overwhelming majority of values of numerical parameters. These results have good applications in real analysis and the theory of partial differential equations. In these domains, along with weighted Lebesgue spaces, general Morrey-type spaces also play an important role.

The classical Morrey space was introduced by Charles Morrey in 1938 in connection with the study of the solution of quasilinear elliptic differential equations. In recent years, questions of measurability and compactness of various operators in spaces of Morrey type have been actively studied.

Global Morrey-type spaces with constant exponents and the boundedness of classical operators in them are well studied in the works of V. I. Burenkov, V. Guliyev, A. Mustafaev, A. Gogotashvili.

The Lebesgue space with a variable exponent was studied in the works of L. Deining, J. Savano.

The Morrey space with variable exponents was introduced in the works of A. Almeida, Zh. Khasanov, S. Samko and the questions of boundedness of classical integral operators in these spaces were considered.

In the works of V. Guliyev, Zh. Khasanov, S. Samko, generalized Morrey spaces with variable exponent were introduced and a condition for boundedness of classical integral operators in these spaces was obtained.

The dissertation introduces global spaces of Morrey type with variable exponents and studies conditions for boundedness of classical integral operators in these spaces.

#### **Purpose of the work.**

- obtaining boundedness conditions for the maximum Hardy-Littlewood operator in variable global Morrey spaces.

- obtaining boundedness conditions for the Riesz potential and its commutator in variable global Morrey spaces.

- obtaining boundedness conditions for the potential of the singular integral and its commutator in variable global Morrey spaces.

- obtaining compactness conditions for sets in variable global Morrey spaces.

- obtaining compactness conditions for the commutator of the Riesz potential in variable global Morrey spaces.

#### **Scientific novelty.**

-boundedness conditions for the maximum Hardy-Littlewood operator in variable global Morrey spaces was obtained.

- boundedness conditions for the Riesz potential and its commutator in variable global Morrey spaces were obtained.

- boundedness conditions for the potential of the singular integral and its commutator in variable global Morrey spaces were obtained.

- conditions for compactness of sets in variable global Morrey spaces were obtained.

- conditions for the compactness of the commutator of the Riesz potential in variable global Morrey spaces were obtained.

### **Research methods.**

The theory of metric functions and methods of functional analysis are used.

### **The practical and theoretical significance of the research.**

The results obtained are used in various studies of functional analysis and solution of equations of mathematical physics.

### **Approbation of the obtained results.**

The main results of the dissertation were discussed at the following conferences and scientific seminars:

- International scientific conference "Modern problems of mathematics and mechanics", dedicated to the 80th anniversary of the birth of Academician V. A. Sadovnichy (Moscow, Moscow State University, May 13-15, 2019);

- International conference "Actual problems of analysis, differential equations and algebra" (EMJ-2019), dedicated to the 10th anniversary of the publication of the "Eurasian Mathematical Journal";

- traditional April mathematical conference (Almaty, April 3-5, 2019);

- International scientific conference "Theoretical and applied problems of mathematics, mechanics and informatics" (June 13, Karaganda, 2019).

### **Publications.**

All scientific results are published in 9 scientific editions (4 articles, 5 theses). In published works in co-authorship with the supervisor, the presentation of the report belongs to the supervisor, and the researcher received the main results of the articles completely independently.

1. Bokayev N. A. Onerbek Zh. M. On the Boundedness of Integral Operators in Morrey-Type Spaces with Variable Exponents // Siberian Advances in Mathematics, 2022, Vol. 32, No. 2, 79-86, DOI: 10.1134/S1055134422020018 (Scopus, процентиль 38)

2. Adilkhanov A.N., Bokayev N. A. Onerbek Zh. M. On the boundedness of the maximal and the Riesz-type potential operators in the global Morrey-type spaces with variable exponent on bounded sets// Kazakh Mathematical Journal. - Institute of

Mathematics and Mathematical Modeling, Almaty, Kazakhstan, 2020. - Vol. 20 no. 3. - P.69-78.

3. Onerbek Zh. M. On the boundedness of the Riesz potential and its commutator's in the global Morrey-type spaces with variable exponents// Vestnik KazNU, -2022. - №2(114), P.54-60

4. Bokayev N.A., Onerbek Zh. M. Calderon-Zigmund integral in the Morrey-type spaces with variable exponents. Vestnik KazNPU. -2022, -No 2(78). -P. 7-13.

### **Structure and volume of the dissertation.**

The work consists of an introduction, two parts, a conclusion and a list of references. The concepts and formulas of numbers consist of three indicators. The first index indicates the number of the section, the second index indicates the number of subsections of the section, the third index indicates the number of statements and formulas in this subsection. The volume of work is 72 pages. The number of links is 50.