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BOUNDARY VALUE PROBLEMS FOR FRACTIONALLY LOADED PARABOLIC EQUATIONS

ABSTRACT

of the dissertation for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100– Mathematics

The relevance of the topic.

To date, loaded heat equations have a wide practical application. In addition, loaded equations constitute a special class of equations with their own specific tasks. You can also note the need to study loaded equations in the study of some inverse problems, in the linearization of nonlinear equations, in the study of some problems of optimal control, etc. In recent years, the volume of research on various boundary value problems for loaded equations has been expanded, the distinguishing feature of which is the presence of a fractional integro-differentiation operator in the boundary conditions or in the equation. Of interest are the boundary value problems for the loaded heat equation, when the loaded term is represented in the form of a fractional derivative. Problems of this kind have not yet been adequately investigated.

The purpose of the work. Statement and study of the solvability of boundary value problems for a fractionally loaded heat equation in certain functional classes; reducing the boundary value problems to Volterra integral equations with kernels containing special functions; study of the solvability of the obtained integral equations, depending both on the order of the fractional derivative in the loaded term of the original boundary value problem, and on the nature of the load behavior.

Research objectives:

1. Give a statement of boundary value problems for a fractionally loaded heat equation and describe the spaces of solutions and given functions;
2. Reduce the formulated boundary value problems to a Volterra integral equation of the second kind with singularities in the kernel or with a kernel containing special functions;
3. Study the limiting cases of the fractional derivative order of the term with the load in the heat equation of the boundary value problem;
4. Establish existence and uniqueness theorems for solutions to problems or related integral equations in certain functional classes, depending both on the intervals of changing the fractional derivative order in the loaded term of the original boundary value problems, and on the nature of the load behavior.

Research object: boundary value problems for fractionally loaded heat equations.

Research subject: solvability of the accompanying Volterra integral equations of the second kind for boundary value problems of heat conduction equations with a loaded term as the form of a fractional derivative.

Research methods. At the first stage of the study, the method of integral

equations is applied, in which the boundary value problem is reduced to solving the corresponding integral equation. Such methods make it possible to formulate boundary value problems more compactly than differential equations, taking into account all conditions of the problem. The problem is reduced to an integral equation by inverting the differential part. When studying the limiting cases of the fractional derivative order in the loaded term of the original problem, methods of the limit theory are used (passage to the limit under the sign of the derivative and / or integral, L'Hopital's rule, etc.). Further, in the study of the integral equation solvability in certain functional classes, methods based on the estimates for the integral operator of the equation are used. Since the kernels of the obtained integral equations contain singularities or special functions, the methods of asymptotic estimates for the integral and series are applied. The same methods are used at the final stage of the study when establishing the dependence of the solution existence and uniqueness for integral equations in certain functional classes on the intervals of changing the fractional derivative order in the loaded term of the original boundary value problems and on the nature of the load movement.

Scientific novelty. The equation of the boundary value problem includes a loaded term in the form of a fractional derivative, and the kernel of the obtaned integral equation contains special functions. Boundary-value problems for the heat equation with a fractional load in certain functional classes, as well as the integral equations accompanying them, which are investigated, are new both in their formulation and in the methods of solving. There are also elements of novelty in the obtained results. Previously, such problems in their entirety were not set and studied..

Theoretical and practical value of the work.

The results of the dissertation are theoretical. A technique has been developed for studying a number of boundary value problems for the heat equation with a loaded term in the form of a fractional derivative, based on reducing the studied problems to Volterra integral equations of the second kind. The study of the integral equation solvability was carried out using the apparatus of special functions.

In addition, the obtained results can serve as a certain contribution to the theory of Volterra type integral equations with kernel singularities. The practical value of the work is determined by the applied significance of Volterra integral equations of the second kind with kernels having various kinds of singularities.

The main provisions for defense.

The following provisions are submitted for defense:

1. Постановки краевых задач для дробно-нагруженного уравнения теплопроводности в функциональных классах;
2. Reducing the boundary value problems to a Volterra integral equation of the second kind with a kernel containing special functions;
3. Investigation of limiting cases of the fractional derivative order of the term with load in the heat conduction equation of a boundary value problem;
4. Determining the change intervals of the fractional derivative order in the loaded term of boundary value problems, when the theorems of solution existence and uniqueness are valid for integral equations, to which the boundary value problems are reduced;

5. Establishment of conditions for the solution existence and uniqueness of integral equations, to which boundary value problems are reduced.

Reliability and validity of the studies are ensured by the constructiveness of the developed and used methods..The auxiliary statements of the problematic issues of each section are formulated in the form of lemmas and statements, and they are strictly proved, and the general ones are in the form of theorems and their proofs are presented in a detailed presentation.

Approbation of the work.

The main results of the dissertation were reported and discussed at the following conferences and seminars:

"Marchuk Scientific Readings - 2020" international conference dedicated to the 95th anniversary of the birth of Academician G.I. Marchuk, Novosibirsk, 2020;

"Modern methods of function theory and related problems", Voronezh Winter Mathematical School (Voronezh, Russia, January 28 - February 2, 2021);

Traditional international April conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 75th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan Tynysbek Sharipovich Kalmenov (IMMM, Almaty, 2021);

at a seminar led by Prof. Jenaliev M.T. (IMMM, Almaty); at a seminar led by Prof. Ramazanov M.I. (Buketov KarU); at the seminar "Fundamentals of fractional calculus" held by prof. Pskhu A.V., director of IAMA KBSC RAS, Nalchik, Russia, ChRes of department of fractional calculus, etc.

The topic of the dissertation research corresponds to the priority direction of development «Scientific research in the field of natural sciences», specialized scientific direction «Fundamental and applied research in mathematics and mechanics». Part of the results of the dissertation were included in the final report on the grant project AP08955795 «Boundary Value Problems for the Heat Equation with Fractional Order Load» (2020-2021).

Publications.

The main results of the dissertation were published in 14 papers: 2 articles - in the journal included in the Scopus list, 5 articles were published in journals recommended by the Committee for Quality Assurance in the sphere Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan and 7 papers - in the materials of international scientific conferences.

In the works performed with co-authors, the contribution of each of the co-authors is equal

The structure and scope of the dissertation.

The 96-page dissertation consists of the following structural elements: designations and abbreviations, introduction, two sections, conclusion, list of reference links, two appendices.

The first section is devoted to the study of an nonhomogeneous boundary value problem for the heat equation with a fractional load in functional classes that are natural for the study (the loaded term is presented as a fractional derivative with respect to the time variable): formulation of the problem, its reduction to the Volterra integral equation of the second kind by representing the problem solution in terms of

the Green function, the study of limiting cases for the fractional derivative order of the term with the load in the equation and the formulation of the main result of the study (establishing the the integral equation solvability depending on the order of the fractional derivative in the loaded term in the equation of the boundary value problem and the nature of the line on which the load is given, for small values of time).

In the second section, we study the solvability of boundary value problems for a fractionally loaded heat equation in the case where the loaded term is represented as a fractional derivative with respect to the spatial variable. The peculiarity of this problem is that, firstly, the loaded term is represented as a fractional derivative with respect to the spatial variable, secondly, the order of the derivative in the loaded term is less than the order of the differential part, and, thirdly, the load point is movable.

Number of used sources – 68.

Keywords. Heat equation, boundary value problems, fractional derivative, order of fractional derivative, loaded term, Volterra equation.