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Boundary value problems of heat conduction in a cone with dynamic boundary conditions

ABSTRACT
of the thesis for the degree of Doctor of Philosophy (PhD)
in the specialty 8D05409201-Mathematics

Relevance of the topic. In the literature, a domain is usually called non-cylindrical if at least one of the parts of its boundary moves with time. If the boundaries of the domain do not change their shape with time, then the domain is called cylindrical. For such domains, the theory of boundary value problems of heat conduction is well developed.

In practice, it often becomes necessary to study the processes of heat and mass transfer in domains of various shapes, the boundaries of which change with time. Problems of this kind arise, for example, in the study of combustion processes in solid fueled rocket engines, the decomposition processes of matter under the influence of temperature, the processes of transferring energy from warm soil to the atmosphere, in the study of nuclear energy problems and the safety of nuclear reactors, ecology, medicine and cryosurgery, natural and artificial process of concrete hardening, as well as in solving some problems of the theory of thermal conductivity of solids (during thermal shock), thermal protection against aerodynamic heating during the movement of spacecraft in dense layers of the atmosphere, etc.

Also, due to the constant increase in the use of contact technology, the problems of the optimal choice of the parameters of contact materials and their modes of operation are actual. Therefore, the study of thermophysical processes occurring in electrical contacts is very relevant in automation, instrumentation, welding technology, electrical equipment and in various devices, where contact elements serve as one of the main links. It has been experimentally established that when the contacts of automatic electric current switches are opened, a liquid-metal bridge appears for a short time, which significantly affects the erosion of the contact material, that is, the contraction effect of the axial section of the arc into a contact spot in the cathode region.

Modeling the thermophysical properties of the bridge, S.N. Kharin came to the boundary value problem in which at the initial moment of contact opening the solution domain is absent. This fact affected the integral equation of the boundary value problem. It turned out that the sequence of Picard approximations of the integral equation is divergent. From a mathematical point of view, the peculiarity of the problem under consideration consists precisely in the presence of a moving boundary and the degeneration of the solution domain at the initial moment of time.

In most papers, the domain in which the solution of the boundary value problem is sought does not degenerate into a point at the initial moment of time. Lions J.L. proposed to solve such problems to use a technique that consists in reducing a non-cylindrical domain to a cylindrical one. In their works, a similar

technique was used by Larkina N.A., Kozhanov A.I., Ferreira J., Benabidallah R., Rivera J., Munoz E., Kheloufi A., Sadallah BK., Cherfaoui S., Kessab A., Cheblakova E.A. and others. There are a number of works, for example, Chapko R., Johansson B.T., Vavrychuk V., Wang Y.F., Huang J., Wen X.X., Dehbozorgi R., Nedaiasl K., where numerical methods are used to solve such problems.

One of the convenient means of solving boundary value problems of heat conduction in domains with variable boundaries are heat potentials. Using them, boundary value problems are reduced to the Volterra type integral equations of the second kind

$$\varphi(t) - \int_0^t K(t,\tau)\varphi(\tau)d\tau = f(t), \qquad (0.1).$$

which are not always solved by the method of successive approximations. If the kernel of equation (0.1) has the following property: the integral of the kernel of the equation does not tend to zero as the upper limit tends to the lower one, then the corresponding integral equations cannot be solved by the method of successive approximations, and in most cases the corresponding homogeneous integral equations have nonzero solutions.

It should also be noted that boundary value problems for spectrally loaded parabolic equations are reduced to similar singular integral equations when the load line moves according to the law $x = t^{\omega}$.

Both in practical applications and theoretically, boundary value problems of heat conduction in domains that degenerate into a point at the initial moment of time are of particular interest. In the general case, the classical methods of mathematical physics are not applicable to this type of problems, since it is not possible to coordinate the solution of the heat equation with the motion of the boundary of the heat transfer domain. Therefore, study boundary value problems in a domain degenerating at the initial moment of time is relevant.

Purpose of research – solvability issues of boundary value problems for heat conduction equations with special boundary conditions in domains that degenerate into a point at the initial moment of time; solution of the singular Volterra type integral equations of the second kind and study of their solvability.

Research objectives:

– formulation of new boundary value problems with special boundary conditions for heat conduction equations in non-cylindrical domains degenerating at the initial moment of time;

– describe the space of solutions and given functions;

– transformation of initial problems;

– reduction of boundary value problems to the singular Volterra type integral equations of the second kind;

– solution of the singular Volterra type integral equations of the second kind, constructing a resolvent;

– solution of initial boundary value problems.

Research object: boundary value problems for equations of parabolic type with special boundary conditions in angular domains that degenerate into a point at the initial moment of time.

Research subject: solvability of boundary value problems for heat conduction equations with time derivatives under boundary conditions in domains that degenerate to a point at the initial moment of time and solution of the related singular Volterra integral equations of the second kind.

Research methodology. The work uses the methods of the general theory of differential equations and functional analysis; methods of Laplace and Fourier integral transformations; special functions; functions of a complex variable.

Scientific novelty. In this work, new boundary value problems for heat conduction equations in degenerate domains with special boundary conditions are solved. The features of the problems under consideration lead to the study of the solvability of the singular Volterra type integral equations of the second kind.

Theoretical and practical value of the work. The results of the thesis are theoretical. It developed a technique for studying a number of new boundary value problems for heat conduction equations in degenerate domains. In addition, the results obtained can serve as a certain contribution to the theory of the Volterra type integral equations with singularities of the kernel, to which the problems under study are reduced.

The practical value of the work is determined by the fact that it is useful in the study of some problems with free boundaries, for example, in the study of the single-phase Stefan problem.

Highlights for defense. The following highlights are defended:

1) solvability of special boundary value problems for heat conduction equations in weighted functional classes;
2) equivalence of boundary value problems to the singular Volterra type integral equations of the second kind found in the work;
3) construction of the resolvent of the singular Volterra type integral equations of the second kind;
4) solvability theorems of the singular Volterra type integral equations of the second kind;
5) solvability theorems of the original boundary value problems.

Reliability and validity of the results obtained in the thesis are confirmed by the constructiveness of the developed and used methods. Auxiliary statements are formulated in the form of lemmas, while the general statements are formulated in the form of theorems, which are strictly proved.

Approbation of the work. Based on the results of the thesis, reports were made at international conferences and at conferences in far abroad countries:

- International Conference “Voronezh Winter Mathematical School” – February, 2021, Voronezh, Russian Federation;
- International Scientific and Practical Conference “Problems of Modern Fundamental and Applied Mathematics” – June 4, 2021, Nur-Sultan, Republic of Kazakhstan;

- Annual International April Mathematical Conference - April 2021, Almaty, Republic of Kazakhstan;
- VI International Scientific Conference “Nonlocal Boundary Value Problems and Related Problems of Mathematical Biology, Informatics and Physics” – December 2021, Nalchik, Russian Federation;
- Annual International April Mathematical Conference in Honor of the Day of Science Workers - April 6-8, 2022, Almaty, Kazakhstan;
- IX International Scientific Conference “Problems of Differential Equations, Analysis and Algebra” – May 24-28, 2022 Aktobe, Kazakhstan;
- at a seminar led by Professor Jenaliyev M.T. (Institute of Mathematics and Mathematical Modeling (IMMM), Almaty, Republic of Kazakhstan);
- at a seminar led by Professor Pskhu A.V. (Institute of Applied Mathematics and Automation (IAMA), Nalchik, Republic of Kabardino-Balkaria, Russian Federation);
- at a seminar led by Professor Ramazanov M.I. (Karaganda Buketov University);
- at the seminar of the Department “Mathematical Analysis and Differential Equations” of the Karaganda Buketov University and others.

Publications. The main results of the dissertation are published in: 5 papers and 7 theses. Of these, 2 papers are in journals with a non-zero IF included in the Scopus database, 3 papers are in journals recommended by the Committee for Quality Assurance in the Sphere of Education and Science of the Ministry of Science and Higher Education of the Republic of Kazakhstan (CQASES MSHE RK). In the papers performed with the co-authors, the main part was done by the author of dissertation, the statements of the selected tasks, the choice of research methods and the final results were discussed with the co-authors.

The topic of the dissertation corresponds to the priority direction of development "Scientific research in the field of natural sciences". The work supported by the grants projects of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan No. AP08956033, 2020–2021, No. AP09259780, 2021-2023.

Structure and scope of the thesis. The dissertation work consists of an introduction, three sections, a conclusion, a list of references and an appendix. The numbering of formulas, theorems, lemmas, remarks in sections is three-digit, the first number means the number of the section, the second - the number of the subsection, the third - the proper number of the formula, theorem, lemma, remark within the subsection.

Number of sources used – 100.

Keywords: noncylindrical domain, cone, boundary value problem of heat conduction, singular Volterra integral equation, Carleman - Vekua regularization method.