ALIBEK ASSETOV

BOUNDARY VALUE PROBLEMS FOR BURGERS EQUATION IN DEGENERATING DOMAINS

ABSTRACT

**of the dissertation for the degree of Doctor of Philosophy (PhD)
in the specialty 8D05409201-Mathematics**

The relevance of the topic. As is known, nonlinear Burgers equations and their modifications are also suitable models of fluid motion in porous media Penetration of the wetting front into a porous medium is a classical problem with a free boundary. Historically, the first and best known example is the Green-Ampt model for water flow in soils. There is a huge variety of situations: chemically reacting media, deformable media, capillarity effects, mass transfer, mixture flows, media with a complex structure, pollution, reclamation, soil freezing, production of composite materials and etc.).

In the works Benia Y. and Sadallah B.-K. were studied the solvability of the boundary value problems for the Burgers equation in a non-rectangular domain

In the literature, it is customary to call an area non-cylindrical if at least one of the parts of its boundary moves with time. If the boundaries of the region do not change their shape with time changes, then the region is called cylindrical. For such areas, the theory of boundary value problems is quite well developed.

In works Benia Y. and Sadallah B.-K. in Sobolev spaces, the correctness of the boundary value problem for the Burgers equation was established. In this case, the domain of independent variables degenerated according to a nonlinear law, and homogeneous Dirichlet conditions were set on the boundary. In Sobolev spaces, by using the Faedo-Galerkin methods and a priori estimates the existence and uniqueness of a regular solution of the boundary value problems under consideration are established.

The paper Solonnikov V.A., Fasano A. studies in the angular domain the boundary value problem for the heat equation with the time derivative under boundary conditions. It is also noted there that the case of an nonhomogeneous boundary value problem "... is useful for study of some problems with free boundaries". For example, for single-phase problem "... Stefan under the following assumptions: the liquid phase with a positive temperature u(x; t) occupies the segment $0 < x < s(t)$, at $x = 0$ a positive heat flow is given, and free boundary $x = s(t)$ starts at the solid boundary $x = 0$, i.e. the conditions(0) = 0 is satisfied". Note that in the paper Solonnikov V.A., Fasano A. the theorem on the unique solvability of the considered boundary value problem in weight Holder spaces is established.

The range of application of boundary value problems for parabolic type equations in a domain with a boundary that varies in time is quite wide. Problems of this kind arise: in the study of thermal processes in electrical contacts, in the

processes of ecology and medicine, in the solution of some problems of hydromechanics, thermomechanics during heat stroke, and so on.

Voluminous literature is devoted to the study of the solvability of linear and nonlinear parabolic equations in cylindrical domains. However, regard to nonlinear boundary value problems in degenerate non-cylindrical domains, they have been studied relatively little.

For angular domains in the Lebesgue classes, we studied boundary value problems of heat conduction and established theorems on their solvability by reducing to the Volterra singular integral equations of the second kind.

We studied various cases of nonhomogeneous boundary. In these cases, it is shown that takes place both unique solvability and non-unique solvability for the corresponding boundary value problems.

Therefore, the study of initial boundary value problems for the Burgers equation with nonlinear and time-derivative boundary conditions in regions degenerating into a point at the initial moment of time is very relevant.

The purpose of the study.
Problems of solvability of boundary value problems for the Burgers equation with special boundary conditions in domains that degenerate into a point at the initial moment of time.

Research objectives:
- formulation of new boundary value problems for the Burgers equation in non-cylindrical degenerate domains with nonlinear and time-derivative boundary conditions;
- solution of the spectral problem;
- formulation and solution of approximate problems;
- solution of a family of auxiliary boundary value problems in quadrangular domains (in the form of a trapezoid);
- a priori estimates for solving problems;
- proof of uniqueness theorems and existence of boundary value problems for the Burgers equation in degenerate domains;
- study of the initial-boundary problem for a Boussinesq-type equation in a non-cylindrical domain.

The objects of research. Boundary value problems for the Burgers and Boussinesq equations with nonlinear and time-derivative boundary conditions in non-cylindrical regions degenerating into a point at the initial time.

Subject of research. Solvability of boundary value problems for the Burgers and Boussinesq equations with nonlinear and time-derivative boundary conditions in regions degenerating into a point at the initial time.

Research methods. The work uses the methods of the general theory of differential values in random derivatives, functional analysis, the method of a priori estimates, Faedo-Galerkin.

Scientific novelty. In contrast to the problems studied so far for the Burgers and Boussinesq equations, this paper proposes formulations and solutions of new boundary value problems, with nonlinear and the presence of a time derivative

boundary conditions for the Burgers and Boussinesq equations in noncylindrical domains that degenerate into a point at the initial time.

Theoretical and practical value of the work. The results of the dissertation are theoretical in nature. It develops methods of a priori estimates and Faedo-Galerkin to study a number of boundary value problems for the Burgers and Boussinesq equations in non-cylindrical domains that degenerate into a point at the initial time.

The practical value of the work is determined by the fact that the Burgers, Boussinesq equations and their modifications are suitable models of fluid motion in porous media, nonlinear thermal fields in high voltage contact devices, nonlinear diffusion processes and the propagation of foreign inclusions in the flows of water and atmospheric areas. In recent years, boundary value problems for these equations have been actively investigated, since the processes occurring in porous media are of particular importance for deep understanding and understanding in the tasks of exploration and effective development of oil and gas fields.

Results presented to the defense. The following provisions are submitted for defense:

1) unique solvability in Sobolev classes of a nonlinear Neumann-type boundary value problem for the Burgers equation in a rectangular domain;

2) 2) solvability theorems in Sobolev classes of a nonlinear boundary value problem for the Burgers equation in a degenerate domain whose degeneracy point is at the origin;

3) theorems on the solvability of a nonlinear boundary value problem for the Burgers equation in a nonlinearly degenerate domain with time derivatives under boundary conditions;

4) sufficient conditions for the well-posedness of a nonlinear boundary value problem for the Burgers equation in a degenerate domain with dynamic boundary conditions;

5) sufficient conditions for unique weak solvability of boundary value problems for the Boussinesq equation in degenerate domains.

The reliability and validity of the studies are ensured by the constructiveness of the developed and used methods. The auxiliary statements of the problematic issues of each section are formulated in the form of lemmas and statements, and they are strictly proved, and the general ones are in the form of theorems and their proofs are presented in a detailed presentation.

Approbation of the dissertation results. The results of the dissertation were tested at international conferences, including conferences abroad, reports were made at seminars: under the guidance of Professor Jenaliyev M.T. (IMMM, Almaty, Republic of Kazakhstan); under the guidance of Professor Pskhu A.V. (Institute of Applied Mathematics and Automation, Nalchik, Kabardino-Balkar Republic, RF); under the guidance of Professor Ramazanov M.I. (KarU named after Academician E.A.Buketov); departments of "Mathematical analysis and differential equation" KarU named after Academician E.A.Buketov and others; at international conferences: "Voronezh Winter Mathematical School" – February 2021 y., Voronezh, RF; The traditional April International Mathematical

Conference in honor of the Day of Science Workers – 6-8 April 2022, Almaty, Republic of Kazakhstan.

Publications. The main results of the dissertation were published in 7 papers: 5 articles and 2 thesis. 1 article - in the journal included in the Scopus list, 4 articles were published in journals recommended by the Committee for Quality Assurance in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan and 2 papers – in the materials of international scientific conferences.

In the works performed with the co-authors, the main part was done by the dissertation student, the statements of the selected tasks, the choice of research methods and the final results were discussed with the co-authors.

The topic of the dissertation research corresponds to the priority development area "Scientific research in natural sciences". The work was carried out within the framework of the grant project of the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan № AP08855372, 2020-2022 yy.

The structure and scope of the dissertation. Dissertation work of 91 pages, consists of an introduction, three sections, a conclusion, a list of sources used and an appendix.

The number of sources used is – 76.

Keywords. Burgers equation, Boussinesq-type equation, boundary conditions, degenerate domain, a priori estimates.