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SOLUTION OF BOUNDARY VALUE PROBLEMS FOR PSEUDOPARABOLIC EQUATIONS OF THE THIRD ORDER

ANNOTATION

of the dissertation for the degree of Doctor of Philosophy(PhD)
at specialty 6D060100- Mathematics

The relevance of the topic.

In this dissertation, linear and nonlinear boundary value problems for pseudo-parabolic equations of the third order with two independent variables are being researched.

The need to study boundary value problems for third-order partial differential equations and to construct approximate methods for finding their solutions arises in the study and modeling of various processes in biology, physics and other sciences.

One of the first studies of differential equations that were not solved with respect to the highest derivative in time is the work of S.L. Sobolev on small oscillations of a rotating fluid, therefore such equations are often called Sobolev-type equations, moreover, in a number of studies the term "pseudo-parabolic equations" is also used.

Pseudo-parabolic equations with various differential operators of the second and higher orders in spatial variables arise in mathematical models of epidemic, diffusion, heat transfer and wave processes. Problems of this kind also arise in the theory of unsteady flow of a viscous gas, in convective diffusion of salts in a porous medium, propagation of initial seals in a viscous gas, moisture transfer in soils, propagation of pulsed radiation waves, in various biological processes, in the theory of inverse problems. In particular, such tasks include filtration of homogeneous liquids in rocks with highly developed fracturing.

The development of computer technology and its comprehensive application in applied tasks imposes new requirements on the methods being developed. Particular attention is paid to methods that differ favorably from others in their constructiveness at the stage of approximate construction of solutions.

One of such methods is the parametrization method suggested` in the works of D.S.Dzhumabaev for solving two-point boundary value problems of ordinary differential equations. The essence of this method is the introduction of additional parameters, such as h-the interval splitting step, η - the number of repeated integrals used in the algorithm and the reduction of the original problem to a multipoint boundary value problem with a parameter. This method allows to establish the condition of solvability of the boundary value problem for ordinary differential equations and to propose a family of algorithms for finding an approximate solution.

In this paper, the parametrization method is developed for semi-periodic boundary value problems for a system of pseudo-parabolic equations of the third order. Based on it, a two-parameter family of algorithms for finding solutions to

semi-periodic boundary value problems in a rectangular domain is constructed.

The importance of the practical application of the theory of semi-periodic boundary value problems for pseudo-parabolic equations in solving various problems of science and technology, the need to expand the class of solvable boundary value problems for pseudo-parabolic equations of the third order and creation of new constructive algorithms for finding their solutions make the topic of the thesis relevant.

The goal of the work. Research of the solvability conditions of linear and nonlinear boundary value problems for pseudo-parabolic equations of the third order and the construction of algorithms for finding its solutions.

Research objectives:

1. Construct algorithms for finding solutions to semi-periodic boundary value problems for linear pseudo-parabolic equations of the third order and obtain conditions for their convergence.

2. To establish coefficient signs of unambiguous solvability of a linear semi-periodic boundary value problem for systems of pseudo-parabolic equations of the third order.

3. To obtain conditions for the existence of an "isolated" solution to a semi-periodic boundary value problem for nonlinear pseudo-parabolic equations on the basis of algorithms for finding solutions to semi-periodic boundary value problems for linear pseudo-parabolic equations of the third order.

Object of the research: boundary value problems for pseudoparabolic equations of the third order.

Subject of the research: algorithms for finding solutions to semi-periodic boundary value problems for linear pseudo-parabolic equations of the third order and conditions for their convergence, coefficient signs of unambiguous solvability of a linear semi-periodic boundary value problem for systems of pseudo-parabolic equations of the third order, conditions for the existence of an "isolated" solution of a semi-periodic boundary value problem for nonlinear pseudo-parabolic equations.

Research methodology.

Methods of the theory of functional analysis, differential equations and mathematical physics are used in the work.

Scientific novelty.

A complex approach to the study and solution of boundary value problems for pseudoparabolic equations of the third order has been developed.

1. Algorithms for finding solutions of semi-periodic boundary value problems for linear pseudo-parabolic equations of the third order are constructed and conditions for their convergence are obtained.

2. Coefficient signs of unambiguous solvability of a linear semi-periodic boundary value problem for systems of pseudo-parabolic equations of the third order are established.

3. Based on algorithms for finding solutions to semi-periodic boundary value problems for linear pseudo-parabolic equations of the third order, the

conditions for the existence of an "isolated" solution of a semi-periodic boundary value problem for nonlinear pseudo-parabolic equations are obtained.

Theoretical and practical value of the work.

The results obtained in this work are of a theoretical nature and can be used in the construction of computational algorithms for solving semi-periodic boundary value problems for systems of pseudo-parabolic equations of the third order, as well as in the reading special courses at mathematical faculties of universities.

Provisions submitted for presentation.

The following are submitted for presentation:

1 Algorithms for finding solutions of a semi-periodic boundary value problem for systems of linear pseudo-parabolic equations of the third order and conditions for their convergence.

2. Coefficient signs of unambiguous solvability of a linear semi-periodic boundary value problem for systems of pseudo-parabolic equations of the third order.

3. Conditions for the existence of an "isolated" solution of a semi-periodic boundary value problem for nonlinear pseudo-parabolic equations obtained on the basis of algorithms for finding solutions of semi-periodic boundary value problems for linear pseudo-parabolic equations of the third order.

The credibility and validity of the conducted researches are ensured by the constructiveness of the methods used. General statements are formulated in the form of theorems and their proofs are presented, auxiliary statements are formulated in the form of lemmas and they are proved.

Approbation of the work.

The main results of the dissertation were reported and discussed at the following conferences and seminars:

- the traditional international April Mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan and the Workshop "Problems of modeling processes in electrical contacts" dedicated to the 80th anniversary of Academician of the National Academy of Sciences of the Republic of Kazakhstan C.N.Kharin (Almaty, April 3-5, 2019);

-International scientific conference "Modern Problems of Mathematics and Mechanics" dedicated to the 80th anniversary of Academician V.A. Sadovnichy (Moscow, 2019);

-International scientific conference "Theoretical and applied problems of mathematics, Mechanics and computer science", dedicated to the 70th anniversary of Ph.D., Professor M.I. Ramazanov (Karaganda, 2019);

-International conference "Actual problems of Analysis, Differential Equations and Algebra (EMJ-2019)" dedicated to the 10th anniversary of the journal "Eurasian Mathematical Jornal" (Nur-Sultan, 2019);

-Uzbek-Russian scientific conference "Non-classical equations of mathematical physics and their applications" (Tashkent, 2019);

- the traditional international April Mathematical conference in honor of the Day of Science Workers of the Republic of Kazakhstan, dedicated to the 1150th

anniversary of Abu Nasir al-Farabi and the 75th anniversary of the Institute of Mathematics and Mathematical Modeling (Almaty, 2020);

-a scientific seminar led by Professor M.I. Ramazanov (Karagandy University named after Academician E.A.Buketov).

Publications.

The main results of the dissertation were published in 10 works: 1 article in a journal included in the Scopus list, 3 articles were published in journals recommended by the Committee for Control in the Field of Education and Science of the Ministry of Education of the Republic of Kazakhstan and 6 works in the materials of international scientific conferences.

In the works performed with co-authors, the contribution of each of the co-authors is equal.

The structure and scope of the dissertation.

The dissertation work with a scope of 85 pages consists of the following structural elements: designations and abbreviations, an introduction, two sections, a conclusion, a list of sources used.

In the first section, the solvability of a linear semi-periodic boundary value problem for a system of pseudo-parabolic equations of the third order and algorithms for finding its solution is investigated. The boundary value problem under research is reduced to a problem consisting of a family of semi-periodic boundary value problems for pseudo-parabolic equations of the third order and functional relations. Based on the parametrization method, coefficient signs of unambiguous and correct solvability of the problem under consideration are obtained.

In the second section, a semi-periodic boundary value problem for pseudoparabolic equations of the third order is investigated. In terms of the initial data, the convergence conditions of the algorithms are established, simultaneously ensuring the existence and isolation of the solution of a nonlinear semi-periodic boundary value problem. Illustrative examples are given.

The number of sources used is 119.

Keywords. Partial differential equations, boundary value problems, pseudo-parabolic equation of the third order, solvability conditions, algorithm, approximate solution.